MODIFIED THERMOSOLUTAL INSTABILITY PROBLEM IN RIVLIN-ERICKSEN ELASTICO-VISCOUS FLUID: A CHARACTERIZATION THEOREM

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Abstract: The present paper considers modified thermosolutal instability problem of a layer of Rivlin-Ericksen elastic-viscous fluid. A mathematical theorem disallowing the existence of neutral or unstable oscillatory motions in an initially bottom heavy modified thermosolutal convection configuration of the Veronis type with a quite general nature of bounding surfaces whenever the modified thermosolutal number is less than a critical value, is established. A similar theorem for Stern's type configuration in initially top heavy modified thermosolutal convection in a layer of Rivlin-Ericksen elastic-viscous fluid is also established.

Keywords: Modified thermosolutal convection; Viscoelastic fluid; Rayleigh Numbers; Lewis Number; Prandtl Number.

1. INTRODUCTION

The thermohaline convection problem has been extensively studied in the recent past on account of its interesting complexities as a double diffusive phenomenon. The study is important because of its direct relevance in many problems of practical interest in the field of oceanography, astrophysics, geophysics, limnology, biomechanics and chemical engineering etc. For a broad and a recent view of the subject one may be referred to Brandt and Fernando [1]. Banerjee et. al. [2] formulated a novel way of combining the governing equations and boundary conditions for each of the Veronis' [3] and Stern's [4] thermohaline configuration and derived a semi- circle theorem prescribing upper limits for complex growth rate of an arbitrary oscillatory perturbation neutral or unstable.

Banerjee et. al [5] in their investigation pointed out that the Rayleigh's utilization of the Boussinesq approximation overlooks a term in the equation of heat conduction. This term finds its place on account of the variations in specific heat at constant volume due to variations in temperature. As a consequence of which, in the usual circumstances it cannot be ignored if the Boussinesq approximation were to be consistently and relatively more accurately applied throughout the analysis. The essential argument on which this term finds a place in the modified theory is this that it is the temperature differences which are of moderate amounts but not necessarily the temperature itself. The incorporation of this term into the calculations adequately completes the qualitative and quantitative gaps in Rayliegh theory.

Theorem 12 and 13 in Banerjee et. al [5] yields in case of Veronis and Stern's thermohaline configurations upper limits for the growth rate of an arbitrary oscillatory perturbation neutral or unstable for the case $\hat{\square}_2 = 0$, which provides natural extension of the earlier results of Banerjee et. al [2] These results are obviously not derivable by the methods adopted by Benerjee et. al when $\hat{\square}_2 \square 0$ on account of non-trivial coupling between $\square, \square$ *and* w in the equation of heat conduction. However, appropriate transformations can overcome this difficulty and can help in deriving the desired results. Mohan [6] extended the results of Banerjee et. al [5] contained in Theorem 12 and 13 for the modified thermohaline convection to the case when $\hat{\square}_2 \square 0$, through the construction of an appropriate transformation on the solution space of the problem and the derivation of suitable integral estimates.

In all the above studies, the fluid has been considered to be Newtonian. However, with the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The Rivlin-Ericksen [7] fluid is such fluid. Many research workers have paid their attention towards the study of Rivlin-Ericksen fluid. Johri [8] has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Sisodia and Gupta [9] and Srivastava and Singh [10] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of the time dependent pressure gradient. Sharma and Kumar [11] have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing effect and introduces oscillatory modes in the system.

Sharma and Kumar [12] have studied the thermal instability in Rivlin-Ericksen elastico-viscous fluid in hydromagnetics.

Motivated by these considerations, the present paper investigates the problem of modified thermosolutal convection in Rivlin–Ericksen viscoelastic fluid of the Veronis' and Stern's type configurations. A mathematical theorem disapproving the existence of neutral or unstable oscillatory motions in an initially bottom heavy modified thermosolutal convection configuration of Veronis type in a layer of Rivlin Ericksen elastic-viscous fluid is established. A similar theorem for Stern's type configuration in initially top heavy modified thermosolutal convection in a layer of Rivlin-Ericksen elastico-viscous fluid is also established.

2. MATHEMATICAL FORMULATION AND ANALYSIS

The relevant governing equations and boundary conditions of modified thermosolutal instability of a Rivlin–Ericksen elastic- viscous fluid are given by [2,7]

$$\left(D^2 - a^2\right)\left[\left(1+\frac{Fp}{\sigma}\right)\left(D^2 - a^2\right) - \frac{p}{\sigma}\right] w = R_T a^2\theta - R_S a^2\phi , \quad (1)$$

$$\left(D^2 - a^2 - p_1\left\langle 1 - T_0\alpha_2\right\rangle\right)\theta - T_0\alpha_2 \hat{R}_3 p\,\phi = -\left(1 - T_0\alpha_2\right)w - T_0\alpha_2 R_3 w , \quad (2)$$

$$\left[D^2 - a^2 - \frac{p}{\tau}\right]\phi = -\frac{w}{\tau} , \quad (3)$$

together with the boundary conditions

$w = 0 = \theta = \phi = Dw$ at z =0 and z =1 (4)

(both boundaries rigid)

or $w = 0 = \theta = \phi = D^2 w$ at z =0 and z =1 (5)

(both boundaries dynamically free)

or $w = 0 = \theta = \phi = Dw$ at z =0

$w = 0 = \theta = \phi = D^2 w$ at z =1 (6)

(lower boundary rigid and upper boundary dynamically free)

or $w = 0 = □ = □ = D^2 w$ at z =0

$w = 0 = □ = □ = Dw$ at z =1 . (7)

(lower boundary dynamically free and upper boundary rigid)

The meanings of symbols from physical point of view are as follows;

z is the vertical coordinate, d/dz is differentiation along the vertical direction, a^2 is square of horizontal wave number, $\sigma = \frac{□}{□}$ is the thermal Prandtl number, $□ = \frac{□_0}{□}$ is the Lewis number, $F = \frac{□_0}{d^2}$ is the viscoelastic parameter, $R_T = \frac{g□□ d^4}{□□}$ (with subscript 1) is the thermal Rayleigh number, $R_S = \frac{g□□ d^4}{□□}$ (with subscript 2) is the concentration Rayleigh number, , w is the vertical velocity, □ is the temperature, □ is the concentration, p is the complex growth rate, $□_2$ is the coefficient of specific heat due to variation in temperature and $□_2$ is analogous coefficient due to variation in concentration.

In equations (1) – (7), z is real independent variable such that $0 \leq z \leq 1$, $D = \frac{d}{dz}$ is differentiation w.r.t z , a^2 is a constant, $\sigma > 0$ is a constant, $□ > 0$ is a constant, 0<F<1, R_T and R_S are positive constants for the Veronis' configuration and negative constants for Stern's configuration, $R_3 = \frac{□□}{□}$ is the ratio of concentration gradient to thermal gradient, $p = p_r + ip_i$ is complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $□(z) = □_r(z) + i□_i(z)$ and $□(z) = □_r(z) + i□_i(z)$ are complex valued functions (and their real and imaginary parts are real valued).

Equations (1) – (3) together with the boundary conditions (4) – (7) describe an eigenvalue problem for p and govern modified thermosolutal instability of Rivlin-Ericksen viscoelastic fluid for any combination of dynamically free and rigid boundaries.

We now prove the following theorems:

Theorem 1: If R > 0, $R_s > 0$, F > 0, $\square\left(1 - T_0\square_2\right) > 1$ $p_r \geq 0$, $p_i \neq 0$, and

$$R'_S \leq \frac{(1 - T_0\alpha_2)}{B}\{\frac{27\pi^4}{4}(1 + \frac{\tau}{\sigma}) + \frac{F\tau}{\sigma}\frac{256\pi^6}{27}\}$$

then a necessary condition for the existence of a non-trivial solution $(w, \square, \square, p)$ of equations (1) – (3) together with boundary conditions (4) – (7) is that

$$R'_S < \frac{F}{B}(1 - T_0\alpha_2).$$

Proof: Equation (2) upon utilizing (3) can be written as

$$\left(D^2 - a^2 - p\left\langle 1 - T\square_0{}_2\right\rangle\right)\square - T\square_0{}_2\,\hat{R}_3\,p\,\square\left(D^2 - a^2\right)\square = -\left(1 - T\square_0\right)w_2 \quad . \tag{8}$$

Using the transformations

$$\tilde{w} = w$$

$$\tilde{\square} = \frac{\left\langle \square\left(1 - T_0\square_2\right) - 1\right\rangle}{T_0\square_2 R_3\square}\square + \square$$

$$\tilde{\square} = \square,$$

(8*)

equations (1), (3) and (8) and the associated boundary conditions (4)-(7) assume the following forms:

$$\left(D^2 - a^2\right)\left[(1 + \frac{Fp}{\square})D^2 - a^2 - \frac{p}{\square}\right]w = R_T a^2\square - R_S a^2\square \quad , \tag{9}$$

$$\left\{D^2 \square a^2 \square p\left(1 \square T \square_0\right)\right\}\square = \square Bw, \tag{10}$$

$$\left[D^2 \square a^2 \square \frac{p}{\square}\right]\square = \square \frac{w}{\square}, \tag{11}$$

with

$$w = 0 = \square = \square = Dw \text{ at z=0 and z=1} \tag{12}$$

or

$$w = 0 = \square = \square = D^2 w \text{ at z=0 and z=1} \tag{13}$$

or

$$w = 0 = \square = \square = Dw \quad \text{at z =0}$$

$$w = 0 = \square = \square = D^2 w \quad \text{at z =1}$$

(14)

or

$$w = 0 = \square = \square = D^2 w \quad \text{at z =0}$$

$$w = 0 = \square = \square = Dw \quad \text{at z =1}$$

(15)

where

$$R_T^{\square} = \frac{R_T T_0 \square_2 R_3 \square}{\left\langle \square\left(1 \square T_0 \square_2\right) \square 1 \right\rangle}, \quad R_S^{\square} = R_S + \frac{R_T T_0 \square_2 R_3 \square}{\left\langle \square\left(1 \square T_0 \square_2\right) \square 1 \right\rangle},$$

and $\quad B = \left(1 \square T_0 \square_2\right)\left[1 + \frac{\left(\square\left\langle 1 \square T_0 \square_2\right\rangle \square 1\right)}{T_0 \square_2 R_3 \square}\right] > 0$

and the symbol ~ has been omitted for convenience.

Multiplying equation (9) by w* (the complex conjugate of w) throughout and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 w^*(D^2-a^2)\left\{\left(1+\frac{Fp}{\varepsilon}\right)(D^2-a^2)-\frac{p}{\varepsilon}\right\}w\,dz = R_T a^2\int_0^1 \theta w^*\,dz - R_S a^2\int_0^1 \phi w^*\,dz. \tag{16}$$

Taking the complex conjugate of equations (10) and (11) and using the resulting equations in equation (9), we get

$$\int_0^1 w^*(D^2-a^2)\left\{\left(1+\frac{Fp}{\varepsilon}\right)(D^2-a^2)-\frac{p}{\varepsilon}\right\}w\,dz = -\frac{R_T a^2}{B}\int_0^1 \theta\left[(D^2-a^2)-p^*\left(1-T_0\theta^2\right)\right]\theta^*\,dz + R_S a^2\int_0^1 \phi\left[k_2(D^2-a^2)-\frac{p^*\sigma}{\varepsilon}\right]\phi^*\,dz. \tag{17}$$

Integrating equation (17) by parts a suitable number of times, using either of the boundary conditions (12)-(15) and one of the following inequalities

$$\int_0^1 \psi^* D^{2n}\psi\,dz = (-1)^n\int_0^1 \left|D^n\psi\right|^2 dz, \tag{18}$$

where,

$\psi = \theta = \phi$, for n = 0, 1 and $\psi = w$, for n = 0, 1, 2,

we have

$$\left(1+\frac{Fp}{\varepsilon}\right)\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz+\frac{p}{\varepsilon}\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz$$
$$=\frac{R_T}{B}a^2\int_0^1\left[\left(\left|D\theta\right|^2+a^2\left|\theta\right|^2\right)+p^*\left(1-T_0\theta^2\right)\left|\theta\right|^2\right]dz$$
$$-R_S a^2\int_0^1\left[\left(\left|D\phi\right|^2+a^2\left|\phi\right|^2\right)+\frac{p^*}{\varepsilon}\left|\phi\right|^2\right]dz \tag{19}$$

Equating the real and imaginary parts of equation (19) equal to zero and using $p_i \neq 0$, we get

$$\int_0^1 \left(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{p_r F}{\varepsilon}\int_0^1 \left(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{p_r}{\varepsilon}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz - \frac{R_T}{B}a^2\int_0^1 \left[\left(|D\theta|^2 + a^2|\theta|^2\right) + p_r\langle 1 - T_0\Phi_2\rangle|\theta|^2\right]dz$$

$$+ R_S a^2 \int_0^1 \left[\left(|D\phi|^2 + a^2|\phi|^2\right) + \frac{p_r}{\varepsilon}|\phi|^2\right]dz = 0$$

(20)

and

$$\frac{F}{\varepsilon}\int_0^1 \left(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{1}{\varepsilon}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz +$$

$$\frac{R_T\left(1 - T_0\Phi_2\right)}{B}a^2\int_0^1 |\theta|^2 dz - R_S a^2\int_0^1 |\phi|^2 dz = 0 \qquad (21)$$

Equation (18) can be written in the alternative form as

$$\int_0^1 \left(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{p_r F}{\varepsilon}\int_0^1 \left(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{p_r}{\varepsilon}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz = \frac{R_T}{B}a^2\int_0^1 \left[\left(|D\theta|^2 + a^2|\theta|^2\right)dz - R_S a^2\int_0^1 \left[\left(|D\phi|^2 + a^2|\phi|^2\right)dz\right.\right.$$

$$+ p_r a^2\int_0^1 \left\{\frac{R_T}{B}\langle 1 - T_0\Phi_2\rangle|\theta|^2 dz - R_S |\phi|^2 dz\right\} \qquad (22)$$

and derive the validity of the theorem from the resulting inequality obtained by replacing each one of terms of this equation by its appropriate estimate.

We first note that since w, Θ and Φ satisfy w (0) = 0 = w (1), Θ (0) = Θ (1) =0 and Φ (0) =0= Φ (1), therefore we have by the Rayleigh-Ritz inequality [13]

$$\int_0^1 |Dw|^2 dz \geq \pi^2 \int_0^1 |w|^2 dz \quad , \tag{23}$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \quad , \tag{24}$$

$$\int_0^1 |D\Phi|^2 dz \geq \pi^2 \int_0^1 |\Phi|^2 dz \quad , \tag{25}$$

and
$$\int_0^1 |D^2 w|^2 dz \geq \pi^4 \int_0^1 |w|^2 dz \; . \tag{26}$$

Utilizing inequalities (23) and (26), we get

$$\int_0^1 \left\{ |D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2 \right\} dz \geq \left(\pi^2 + a^2\right)^2 \int_0^1 |w|^2 dz \; . \tag{27}$$

Further, since $p_r \geq 0$, therefore we have

$$\frac{p_r F}{\varepsilon} \int_0^1 \left\{ |D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2 \right\} dz \geq 0 \quad , \tag{28}$$

and

$$\frac{p_r}{\varepsilon} \int_0^1 \left(|Dw|^2 + a^2 |Dw|^2 \right) dz \geq 0 \quad . \tag{29}$$

Now, multiplying equation (10) by Θ^* (the complex conjugate of Θ) and integrating the various terms on the left hand side of the resulting equation by parts for an appropriate number of times by making use of the boundary conditions on Θ namely $\Theta(0) = 0 = \Theta(1)$, we have from the real part of the final equation

$$\int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz + p_r (1 - T_0) \int_0^1 |\Theta|^2 dz = \text{Real part of} \left\{ B \int_0^1 \Theta^* w dz \right\}$$

$$\leq B \left| \int_0^1 \Theta^* w dz \right|$$

$$\leq B \int_0^1 |\Theta^* w| dz$$

$$\leq B \int_0^1 |\Theta| |w| dz$$

$$\leq B \left(\int_0^1 |\Theta|^2 dz \right)^{1/2} \cdot \left(\int_0^1 |w|^2 dz \right)^{1/2}.$$

(using Schwartz inequality)

Using inequality (22) and the fact that $p_r \geq 0$, in the above inequality, we have

$$\left(\pi^2 + a^2\right) \int_0^1 |\Theta|^2 dz \leq B \left(\int_0^1 |\Theta| |w| dz \right)^{1/2} \cdot \left(\int_0^1 |w|^2 dz \right)^{1/2},$$

which implies that

$$\left(\int_0^1 |\Theta|^2 dz \right)^{1/2} \leq B \frac{1}{\left(\pi^2 + a^2\right)} \left(\int_0^1 |w|^2 dz \right)^{1/2},$$

and thus

$$\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz \le B\frac{1}{\left(\pi^2 + a^2\right)}\int_0^1 |w|^2 dz \quad . \tag{30}$$

Further, using inequality (25), we have

$$\int_0^1 \left(|D\Phi|^2 + a^2|\Phi|^2\right)dz \ge \left(\pi^2 + a^2\right)\int_0^1 |\Phi|^2 dz \quad . \tag{31}$$

It follows from equation (21), that

$$R_s' a^2 \int_0^1 |\Phi|^2 dz \le \frac{F}{\varepsilon}\int_0^1 \left(|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{1}{\varepsilon}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz \tag{32}$$

Combining the inequalities (31) and (32), we have

$$\int_0^1 \left(|D\Phi|^2 + a^2|\Phi|^2\right)dz \le \frac{\left(\pi^2 + a^2\right)F}{\varepsilon R_s a^2}\int_0^1 \left\{|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right\}dz + \frac{\left(\pi^2 + a^2\right)}{\varepsilon R_s a^2}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz \; .$$

Using inequalities (23) and (26) in the above inequality, we get

$$\int_0^1 \left(|D\Phi|^2 + a^2|\Phi|^2\right)dz \le \frac{\left(\pi^2 + a^2\right)^3 F}{\varepsilon R_s a^2}\int_0^1 |w|^2 dz + \frac{\left(\pi^2 + a^2\right)^2}{\varepsilon R_s a^2}\int_0^1 |w|^2 dz \quad . \tag{33}$$

Also, from equation (22) and the fact that $p_r \ge 0$, we obtain,

$$p_r a^2 \int_0^1 \left\{\frac{R_T}{B}\left\langle 1 - T_0 \right\rangle |\Theta_2|^2 dz - R_s\int_0^1 |\Phi|^2 dz\right\} \ge 0 \tag{34}$$

Now if permissible, let

$$R'_S \geq \frac{R'_T}{B}(1 - T_0\alpha_2).$$

Then, in that case, we derive from equation (22) and inequalities (27) – (34), that

$$[(\pi^2 - a^2)\{1 + \frac{\tau}{\sigma} + \frac{(\pi^2 + a^2)F\tau}{\sigma}\} - \frac{R'_S a^2 B}{(\pi^2 + a^2)(1 - T_0\alpha_2)}]\int_0^1 |w|^2 dz < 0 \tag{35}$$

which implies that

$$[(\pi^2 - a^2)\{1 + \frac{\tau}{\sigma} + \frac{(\pi^2+a^2)F\tau}{\sigma}\} - \frac{R'_S a^2 B}{(\pi^2+a^2)(1-T_0\alpha_2)}]\int_0^1 |w|^2 dz < 0,$$

and thus we necessarily have

$$R'_S > \frac{(1 - T_0\alpha_2)}{B}\{\frac{27\pi^4}{4}(1 + \frac{\tau}{\sigma}) + \frac{F\tau}{\sigma}\frac{256\pi^6}{27}\}$$

since the minimum values of $\frac{(\pi^2 + a^2)^3}{a^2}$ and $\frac{(\pi^2 + a^2)^4}{a^2}$ for $a^2 > 0$ are $\frac{27\pi^4}{4}$ and $\frac{256\pi^6}{27}$ respectively .

Hence, if

$$R'_S \leq \frac{(1 - T_0\alpha_2)}{B}\{\frac{27\pi^4}{4}(1 + \frac{\tau}{\sigma}) + \frac{F\tau}{\sigma}\frac{256\pi^6}{27}\}$$

then we must have

$$R'_S < \frac{R'_T}{B}(1 - T_0\alpha_2).$$

and this completes the proof of the theorem.

Theorem 1 implies from the physical point of view that the modified thermosolutal convection of the Veronis' type in the Rivlin-Ericksen elastico-viscous fluid cannot manifest as an oscillatory motions of growing amplitude in an initially bottom heavy configuration if

$$R'_S \leq \frac{(1 - T_0\alpha_2)}{B}\{\frac{27\pi^4}{4}(1 + \frac{\tau}{\sigma}) + \frac{F\tau}{\sigma}\frac{256\pi^6}{27}\}$$

Further this result is uniformly valid for the quite general nature of the bounding surfaces.

SPECIAL CASE 1: For the case when F = 0 (Newtonian Fluid) Theorem 1 can be restated as:

Theorem 1: If $R > 0$, $R_S > 0$, $p_r \geq 0$, $p_i \neq o$ and,

$$R'_S \leq \frac{(1 - T_0\alpha_2)}{B}\{\frac{27\pi^4}{4}(1 + \frac{\tau}{\sigma})\}$$

then a necessary condition for the existence of a non-trivial solution (w, □, □, p) of equations (1) – (3) together with boundary conditions (4) – (7) is that:

$$R'_S < \frac{E}{B}(1 - T_0\alpha_2).$$

Theorem 2: If $R < 0$, $R_S < 0$, $F > 0$, $p_r \geq 0$, $p_i \neq 0$, □(1 □ T_0□ 2) > 1

$$|R'_T| \leq \frac{B}{(1 - T_0\alpha_2)}\{\frac{27\pi^4}{4}(1 + \frac{1}{\sigma}) + \frac{F}{\sigma}\frac{256\pi^6}{27}\}$$

then a necessary condition for the existence of a non-trivial solution (w, □, □, p) of equation (9) – (11) together with boundary conditions (12)-(15) is that

$$|R'_T| < \frac{|R'_S|B}{(1 - T_0\alpha_2)}.$$

Proof: Putting $R_T' = -|R_T'|$, $R_S' = -|R_s'|$ in equation (9) and proceeding exactly as in Theorem 1, the desired result follows.

From the physical point of view, **Theorem 2** implies that the modified thermosolutal convection of the Stern's type in the Rivlin-Ericksen elastico-viscous fluid cannot manifest as an oscillatory motions of growing amplitude in an initially top heavy configuration if

$$|R_T'| \leq \frac{B}{(1 - T_0\alpha_2)}\{\frac{27\pi^4}{4}(1 + \frac{1}{\sigma}) + \frac{F}{\sigma}\frac{256\pi^6}{27}\}$$

Further this result is uniformly valid for the quite general nature of the bounding surfaces.

SPECIAL CASE 2: For the case when F = 0 (Newtonian Fluid) Theorem 2 can be restated as:

Theorem 2: If R< 0, R_S< 0, $p_r \geq 0$, p_i □ 0 and

$$|R_T'| \leq \frac{B}{(1 - T_0\alpha_2)}\{\frac{27\pi^4}{4}(1 + \frac{1}{\sigma})\},$$

then a necessary condition for the existence of a non-trivial solution (w, □ , □ , p) of equations (9) – (11) together with boundary conditions (12) – (15) is that

$$|R_T'| < \frac{|R_S^F|B}{(1-T_0\alpha_2)}.$$

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