NEW COLORING FOR BLOCKS – AUM BLOCK COLORING FOR STANDARD GRAPHS

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Abstract
In this paper, we introduce the new concept of block coloring for graphs. Different kinds of coloring such as coloring of vertices[1], coloring of edges [2] and coloring of faces[3] have already been defined for various types of graphs. As the block graphs find enumerable applicationsin network theory,we introduce the theory of coloring of blocks in graphs, named as block coloring and study on blocking coloring of specific graphs.The main objective of this paper is to introduce coloring of blocks and explicitly define block coloring for path, cycle, complete graph,n −barbell graph, windmill graph, friendship graph, cactus graph, bipartite graph and their extensions, with suitable examples.

Keywords: Block coloring, AUM block coloring, path,friendship graph, cactus graph.

1. INTRODUCTION

In 1852 Graph coloring was used by Francis Guthrie to color the map of countries of England and found only four colors were required. Thus initiated the theory of graph coloring[3]. Vertex coloring was introduced by Brooks, R. in 1941 [1]. Edge coloring of the graph was introduced by Andersen, Lars Døvling in 1977 [2]. There are several coloring of graphs studied across the globe namely domianator coloring [4], Total dominator coloing [5], power dominator coloring [[6] – [9]], Rainbow dominator coloring [10] etc.

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In this paper, we define the block coloringfind AUM block chromatic number for path, cycle, complete graph windmill graph, friendship graph, cactus graph, bipartite graph. Suitable examples are given for each labeling on the specified graphs. Throughout this paper, let consider thegraph G, which is simple, finite, connected, $G = (V(G), E(G), B(G))$, $|V(G)| = n$, $|E(G)| = m$, $|B(G)| = l$.

2. PRELIMINARIES

Definition 1[11]: In a graph, coloring is an assignment of colors to the vertices or edges or both subject to certain condition(s).

Definition 2[11]:A graph is a block graph if every block (maximal 2-connected component) is a clique. If G is any undirected graph, the block graph of G, denoted by B(G) is a non separable maximal subgraph of the graph. It is clear that any two blocks of a graph have at most one vertex in common.

3. BLOCK CLORING

Let G be a graph with n vertices, m edges and l blocks, $p, q, l \geq 1$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$, $E(G) = \{e_1, e_2, \ldots, e_m\}$, $B(G) = \{B_1, B_2, \ldots, B_l\}$ denote the vertex set, edge set and the block set of G respectively.

Definition 5: AUM Block Coloring
AUM block coloring of a graph G is assignment of colors to the blocks of G.

Definition 6: Proper AUM Block Coloring
AUM block coloring of G is proper if different colors are assigned to the blocks that have a common vertex**.**

The minimum number of colors required for proper AUM block coloring of the graph G, is called AUM block chromatic number. It is denoted as χ_{Bl}

3.1 BLOCK COLORING OF STANDARD GRAPHS WITH SINGLE BLOCK

First, we consider the standard graphs with single block.

Proposition 1: For $n \geq 3$, the AUM block chromatic number of cycle, C_n is 1.
Proof: We consider cycle $c_n$with $n \geq 3$ nodes. We prove by induction that the cycle with n nodes will have only one block. Therefore the AUM block chromatic number is one.
Case (i) Let $n = 3$
Let C_3 be the cycle with 3 nodes. The vertices be v_1, v_2, v_3, edges be $e_1 = \{v_1v_2\}, e_2 = \{v_2v_3\}, e_3 = \{v_3v_1\}$. Since the cycle is a closed walk, it has only one block B_1. This block is colored with color c_1. The AUM block chromatic number of cycle c_3 is 1.
Case (ii) Let $n = 4$
Let C_4 be the cycle with 4 nodes. The vertices be v_1, v_2, v_3, v_4 edges be $e_1 = \{v_1v_2\}, e_2 = \{v_2v_3\}, e_3 = \{v_3v_4\}, e_4 = \{v_4v_1\}$. Since the cycle is a closed walk, it has only one block B_1. This block is colored with color c_1. The AUM block chromatic number of the cycle c_4 is1.

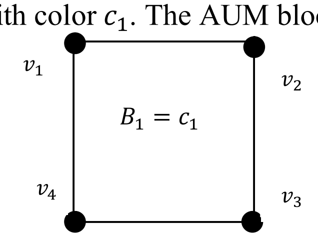

Fig 1, the cycle

Case (iii) Let $n = k - 1$
Let C_{k-1} be the cycle with $k - 1$ nodes. The vertices be $v_1, v_2, v_3, v_4, \ldots, v_{k-1}$ edges be $e_1 = \{v_1v_2\}, e_2 = \{v_2v_3\}, e_3 = \{v_3v_4\}, e_4 = \{v_4v_5\}, \ldots, e_{k-1} = \{v_{k-1}v_1\}$. Since the cycle

$c_{k-1}$is a closed walk, it has only one block B_1. This AUM block is colored with color c_1. The block chromatic number of the cycle c_{k-1} is 1.

Case (iv) Let $n = k$

Let C_{k-1} be the cycle with k nodes. The vertices be $v_1, v_2, v_3, v_4, \dots, v_k$ edges be $e_1 = \{v_1v_2\}, e_2 = \{v_2v_3\}, e_3 = \{v_3v_4\}, e_4 = \{v_4v_5\}, \dots, e_k = \{v_kv_1\}$. Since the cycle c_k is a closed walk, it has only one block B_1. This block is colored with color c_1. The AUM block chromatic number of the cycle c_k is 1.

Proposition 2: For $n \geq 2$, the AUM block chromatic number of complete graph K_n is 1.
Let the complete graph be K_n with n vertices and m edges. Let $v_1, v_2, v_3, \dots, v_n$ be the n vertices. Since complete graph is the maximal connected graph, then complete graph has single block for all $n \geq 3$. Then the AUM block chromatic number of complete graph is 1.

Proposition 3: For $n \geq 3$, the AUM block chromatic number of wheel graph, K_n is 1.
The proof follows from proposition 2 and proposition 1.

Proposition 4: For $n \geq 3$, the AUM block chromatic number of grid graph P_nx$P_m$is 1.
The proof follows from proposition 2 and proposition 1.

Proposition 5: For $n \geq 3$, the AUM block chromatic number of gear graph $G_r$is 1.
The proof follows from proposition 2 and proposition 1.

Proposition 6: For $n \geq 3$, the AUM block chromatic number of complete bipartite graph$K_{n,n}$is 1.
The proof follows from proposition 2 and proposition 1.

Proposition 7: For $n \geq 3$, the AUM block chromatic number of fan graph $F_n$is 1.
The proof follows from proposition 2 and proposition 1.
Remark 6: For any graph that has only one block, AUM block coloring is 1.

3.2 BLOCK COLORING OF STANDARD GRAPHS

We find block chromatic number of some standard graphs with $l \geq 2$ blocks

Theorem 7: Every path $P_n, n \geq 3$the AUM block chromatic number is 2.
Proof: Let $P_n, n \geq 3$ be the path graph. Let $V(G) = \{v_1, v_2, \dots, v_n\}$, $E(G) = \{e_1, e_2, \dots, e_{n-1}\}$, $B(G) = \{B_1, B_2, \dots, B_{n-1}\}$ denote the vertex set, edge set and the block set of P_n.$|V(G)| = n$, $|E(G)| = n-1, |B(G)| = n-1$. Based on the vertices on the path P_n we have following cases.

Case(i): Let $n \geq 3$&n is odd,

Let $P_n, n \geq 3$ be the path graph. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$, $E(G) = \{e_1, e_2, \ldots, e_{n-1}\}$, $B(G) = \{B_1, B_2, \ldots, B_{n-1}\}$ denote the vertex set, edge set and the block set of P_n.$|V(G)| = n$, $|E(G)| = \text{n} - 1, |B(G)| = n - 1$.

Assign the color c_1 to odd indexed blocks the path $P_n\{B_1, B_3, B_5.., B_n\}$. Color c_2 is assigned to the even indexed blocks of the path $P_n\{B_2, B_4, \ldots, B_{n-1}\}$. This block coloring is proper. The AUM block chromatic number for the path$P_n, n \geq 3$ is 2. i.e., $\chi_{Bl}(P_n) = 2$.

Case(ii): Let $n \geq 4 \& n$ is even,
Let $P_n, n \geq 4$ be the path graph. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$, $E(G) = \{e_1, e_2, \ldots, e_{n-1}\}$, $B(G) = \{B_1, B_2, \ldots, B_{n-1}\}$ denote the vertex set, edge set and the block set of P_n.$|V(G)| = n$, $|E(G)| = \text{n} - 1, |B(G)| = n - 1$.

Assign the color c_1 to odd indexed blocks the path $P_n\{B_1, B_3, B_5.., B_{n-1}\}$. Color c_2 is assigned to the even indexed blocks of the path $P_n\{B_2, B_4, \ldots, B_n\}$. This block coloring is proper. The AUM block chromatic number for the path $P_n, n \geq 3$ is 2. i.e., $\chi_{Bl}(P_n) = 2$.

Example 8: In the fig. 2the AUM block coloring of path P_5

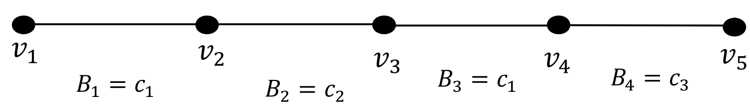

Fig. 2 path P_5

Theorem 2: The n −barbell graph $B(K_n, K_n), n \geq 2$ the AUM block chromatic number is 2.
Proof: Let $B(K_n, K_n), n \geq 2$be the n −barbell graph. Let $V(G) = \{u_1, u_2, \ldots, u_n\} \cup \{v_1, v_2, \ldots, v_n\}$, $E(G) = \{e_1, e_2, \ldots, e_{n(n-1)+1}\}$, $B(G) = \{B_1, B_2, \text{B}_3\}$ denote the vertex set, edge set and the block set of $\text{B}(\text{K}_\text{n}, K_n)$. $|V(G)| = n$, $|E(G)| = n(n-1) + 1$, $|B(G)| = 3$.

Assign the color $c_1$to the blocks $B_1$and B_3.The color$c_2$be assigned to the block B_2.This block coloring is proper. The AUM block chromatic number for the n −barbell graph $B(K_n, K_n)$, is 2. i.e., $\chi_{Bl}\big(B(K_n, K_n)\big) = 2$.

In the fig. 3the block coloring of 3 −barbell graph $B(K_3, K_3)$

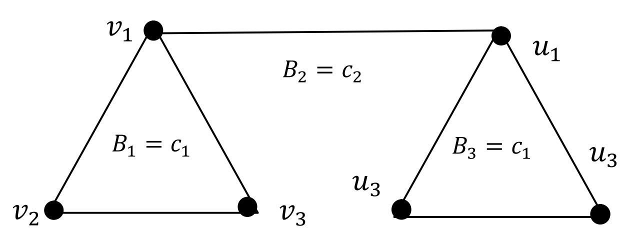

Fig. 3 3 −barbell graph $B(K_3, K_3)$

Theorem 3: The windmill graph $Wd(k,n)), n \geq 2, k \geq 2$ the AUM block chromatic number is n

Proof: Let $Wd(k,n), k \geq 2, n \geq 2$be the windmill graph. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1, v_3^1 \dots, v_{k-1}^1\} \cup \{v_1^2, v_2^2, v_3^2 \dots, v_{k-1}^2\} \cup \{v_1^3, v_2^3, v_3^3 \dots, v_{k-1}^3\} \cup \dots \cup \{v_1^n, v_2^n, v_3^2 \dots, v_{k-1}^2\}$, $E(G) = \{e_1, e_2, \dots, e_l\}$, $B(G) = \{B_1, B_2, B_3, \dots, B_n\}$ denote the vertex set, edge set and the block set of $Wd(k,n), k \geq 2, n \geq 2$. $|V(G)| = (k-1)^n + 1$, $|B(G)| = n$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ and B_3. This block coloring is proper. The AUM block chromatic number for the windmill graph $Wd(k,n))k \geq 2, n \geq 2$ is n. i.e., $\chi_{Bl}(Wd(k,n))) = n$.

In the fig. 4 the AUM block coloring of windmill graph $Wd(4,4)$

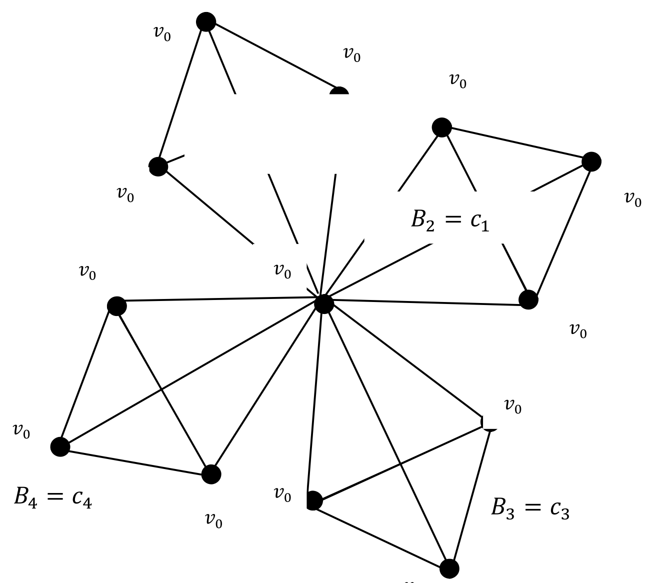

Fig 4: Windmill$Wd(4,4)$

Theorem 4: The friendship graph $F_3^n, n \geq 2$, the AUM block chromatic number is n.

Proof: Let $F_3^n, n \geq 2$,be the friendship graph. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1\} \cup \{v_1^2, v_2^2\} \cup \{v_1^3, v_2^3\} \cup \dots \cup \{v_1^n, v_2^n\}$,$E(G) = \{e_1, e_2, \dots, e_l\}$, $B(G) = \{B_1, B_2, B_3, \dots, B_n\}$ denote the vertex set, edge set and the block set of friendship graph $F_3^n, n \geq 2$. $|V(G)| = (2)^n + 1$, $|B(G)| = n$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$. This block coloring is proper. The AUM block chromatic number for the friendship graph $F_3^n, n \geq 2$ is n. i.e., $\chi_{Bl}(F_3^n) = n$.

Theorem 5: The cactus graph $C_k^n, n \geq 2, k \geq 4$ the AUM block chromatic number is n.

Proof: Let $C_k^n, n \geq 2, k \geq 3$be the cactus graph. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1, v_3^1 \dots, v_{k-1}^1\} \cup \{v_1^2, v_2^2, v_3^2 \dots, v_{k-1}^2\} \cup \{v_1^3, v_2^3, v_3^3 \dots, v_{k-1}^3\} \cup \dots \cup \{v_1^n, v_2^n, v_3^n \dots, v_{k-1}^n\}$,$E(G) =$

$\{e_1, e_2, \ldots, e_l\}$, $B(G) = \{B_1, B_2, B_3, \ldots, B_n\}$ denote the vertex set, edge set and the block set of $C_k^n, n \geq 2, k \geq 4$. $|V(G)| = (k-1)^n + 1$, $|B(G)| = n$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$. This block coloring is proper. The AUM block chromatic number for the cactus graph $C_k^n, n \geq 2, k \geq 4$ is n. i.e., $\chi_{Bl}(C_k^n) = n$.

Theorem 6: The two copies of friendship graph $F_3^n, n \geq 2$,joined by the path union ofP_2, the AUM block chromatic number of G is n.

Proof: Let $F_3^n, n \geq 2$,be the friendship graph. Considering two copies of friendship graphs F_3^nconnected with the 2 vertices of the path P_2. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1\} \cup \{v_1^2, v_2^2\} \cup \{v_1^3, v_2^3\} \cup \ldots \cup \{v_1^n, v_2^n\} \cup \{u_0\} \cup \{u_1^1, u_2^1\} \cup \{u_1^2, u_2^2\} \cup \{u_1^3, u_2^3\} \cup \ldots \cup \{u_1^n, u_2^n\}$,$B(G) = \{B_1, B_2, B_3, \ldots, B_{2n+1}\}$ denote the vertex set, and the block set of friendship graph $F_3^n, n \geq 2$. $|V(G)| = 2(2)^n + 2$, $|B(G)| = 2n + 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first friendship graph. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of Second friendship graph respectively. Block B_{2n+1} is colored with the color which is not colored to the blocks B_1&B_{n+1} This block coloring is proper. The AUM block chromatic number for the two copies of friendship graph $F_3^n, n \geq 2$,joined by the path union of$P_2$is $n+1$. i.e., $\chi_{Bl}(G) = n$.

Theorem 7: The two copies of cactus graph $C_k^n, n \geq 2, k \geq 4$,joined by the path union ofP_2, the AUM block chromatic number of Gis$n$.

Proof: Let $C_k^n, n \geq 2, k \geq 3$be the cactus graph. Considering two copies of cactus graph $C_k^n, n \geq 2, k \geq 4$connected with the 2 vertices of the path P_2. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1, v_3^1 \ldots, v_{k-1}^1\} \cup \{v_1^2, v_2^2, v_3^2 \ldots, v_{k-1}^2\} \cup \{v_1^3, v_2^3, v_3^3 \ldots, v_{k-1}^3\} \cup \ldots \cup \{v_1^n, v_2^n, v_3^n \ldots, v_{k-1}^n\} \cup \{u_0\} \cup \{u_1^1, u_2^1, u_3^1 \ldots, u_{k-1}^1\} \cup \{u_1^2, u_2^2, u_3^2 \ldots, u_{k-1}^2\} \cup \{u_1^3, u_2^3, u_3^3 \ldots, u_{k-1}^3\} \cup \ldots \cup \{u_1^n, u_2^n, u_3^n \ldots, u_{k-1}^n\}$,, $B(G) = \{B_1, B_2, B_3, \ldots, B_{2n+1}\}$ denote the vertex set, and the block set of $C_k^n, n \geq 2, k \geq 4$. $|V(G)| = 2(k-1)^n + 2$, $|B(G)| = 2n + 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first cactus graph $C_k^n, n \geq 2, k \geq 4$. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of Second cactus graph $C_k^n, n \geq 2, k \geq 4$ respectively. Block B_{2n+1} is colored with the color which is not colored to the blocks B_1&B_{n+1}. This block coloring is proper. The AUM block chromatic number for the two copies of cactus graph $C_k^n, n \geq 2, k \geq 4$,joined by the path union of$P_2$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 8: The two copies of graph windmill graph $Wd(k,n)), n \geq 2, k \geq 2$,joined by the path union ofP_2, the AUM block chromatic number of Gis$n$.

Proof: Let $Wd(k,n)), n \geq 2, k \geq 2$be the windmill graph. Considering two copies of windmill graph $Wd(k,n)), n \geq 2, k \geq 2$connected with the vertices of the path P_2. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1, v_3^1 \dots, v_{k-1}^1\} \cup \{v_1^2, v_2^2, v_3^2 \dots, v_{k-1}^2\} \cup \{v_1^3, v_2^3, v_3^3 \dots, v_{k-1}^3\} \cup \dots \cup \{v_1^n, v_2^n, v_3^n \dots, v_{k-1}^n\} \cup \{u_0\} \cup \{u_1^1, u_2^1, u_3^1 \dots, u_{k-1}^1\} \cup \{u_1^2, u_2^2, u_3^2 \dots, u_{k-1}^2\} \cup \{u_1^3, u_2^3, u_3^3 \dots, u_{k-1}^3\} \cup \dots \cup \{u_1^n, u_2^n, u_3^n \dots, u_{k-1}^n\}$, $B(G) = \{B_1, B_2, B_3, \dots, B_{2n+1}\}$ denote the vertex set and the block set of $Wd(k,n)), n \geq 2, k \geq 2$. $|V(G)| = 2(k-1)^n + 2$, $|B(G)| = 2n + 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first windmill $Wd(k,n)), n \geq 2, k \geq 2$. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of second windmill graph$Wd(k,n)), n \geq 2, k \geq 2$ respectively. Block B_{2n+1} is colored with the color which is not colored to the blocks B_1&B_{n+1} . This block coloring is proper. The AUM block chromatic number for the two copies of windmill $Wd(k,n)), n \geq 2, k \geq 2$,joined by the path union of$P_2$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 9: The three copies of friendship graph $F_3^n, n \geq 2$,joined by the path union ofP_3, the AUM block chromatic number of G is n.

Proof: Let $F_3^n, n \geq 2$,be the friendship graph. Considering three copies of friendship graphs F_3^nconnected with the vertices of the path P_3. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1\} \cup \{v_1^2, v_2^2\} \cup \{v_1^3, v_2^3\} \cup \dots \cup \{v_1^n, v_2^n\} \cup \{u_0\} \cup \{u_1^1, u_2^1\} \cup \{u_1^2, u_2^2\} \cup \{u_1^3, u_2^3\} \cup \dots \cup \{u_1^n, u_2^n\} \cup \{w_0\} \cup \{w_1^1, w_2^1\} \cup \{w_1^2, w_2^2\} \cup \{w_1^3, w_2^3\} \cup \dots \cup \{w_1^n, w_2^n\}$,$B(G) = \{B_1, B_2, B_3, \dots, B_{2n+1}\}$ denote the vertex set and the block set of three copies of friendship graph $F_3^n, n \geq 2$. $|V(G)| = 3(2)^n + 3$, $|B(G)| = 3n + 2$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first friendship graph. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of Second friendship graph respectively. Colors $c_i 1 \leq i \leq n$to the blocks $B_k$for $2n+1 \leq k \leq 3n$ of third friendship graph.

Blocks B_{3n+1}&$B_{3n+2}$are colored with the colors which are not colored to the blocks B_1&B_{n+1}&B_{2n+1}. This block coloring is proper. The AUM block chromatic number for the three copies of friendship graph $F_3^n, n \geq 2$,joined by the path union of$P_3$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 10: The three copies of cactus graph $C_k^n, n \geq 2, k \geq 4$,joined by the path union ofP_3, the AUM block chromatic number of Gis$n$.

Proof: Let $C_k^n, n \geq 2, k \geq 3$be the cactus graph. Considering three cactus graph $C_k^n, n \geq 2, k \geq 4$connected with the vertices of the path P_3. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1, v_3^1 \dots, v_{k-1}^1\} \cup \{v_1^2, v_2^2, v_3^2 \dots, v_{k-1}^2\} \cup \{v_1^3, v_2^3, v_3^3 \dots, v_{k-1}^3\} \cup \dots \cup \{v_1^n, v_2^n, v_3^n \dots, v_{k-1}^n\} \cup \{u_0\} \cup \{u_1^1, u_2^1, u_3^1 \dots, u_{k-1}^1\} \cup \{u_1^2, u_2^2, u_3^2 \dots, u_{k-1}^2\} \cup \{u_1^3, u_2^3, u_3^3 \dots, u_{k-1}^3\} \cup \dots \cup \{u_1^n, u_2^n, u_3^n \dots, u_{k-1}^n\} \cup \{w_0\} \cup \{w_1^1, w_2^1, w_3^1 \dots, w_{k-1}^1\} \cup \{w_1^2, w_2^2, w_3^2 \dots, w_{k-1}^2\} \cup$

$\{w_1^3, w_2^3, w_3^3 \dots, w_{k-1}^3\} \cup \dots \cup \{w_1^n, w_2^n, w_3^n \dots, w_{k-1}^n\}$, $B(G) = \{B_1, B_2, B_3, \dots, B_{3n+2}\}$ denote the vertex set and the block set of three copies of cactus $C_k^n, n \geq 2, k \geq 4$. $|V(G)| = 2(k-1)^n + 2$, $|B(G)| = 2n + 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first cactus graph. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of Second cactus graph.Colors $c_i 1 \leq i \leq n$to the blocks $B_k$for $2n+1 \leq k \leq 3n$ of third cactus graph . Blocks B_{3n+1}&$B_{3n+2}$are colored with the colors which are not colored to the blocks B_1&B_{n+1}&B_{2n+1}. This block coloring is proper. The AUM block chromatic number for the three copies of cactus graph $C_k^n, n \geq 2, k \geq 4$,joined by the path union of$P_3$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 11: The three copies of graph windmill graph $Wd(k,n)), n \geq 2, k \geq 2$,joined by the path union ofP_3, the AUM block chromatic number of Gis$n$.

Proof: Let $Wd(k,n)), n \geq 2, k \geq 2$be the windmill graph. Considering three copies of windmill graph $Wd(k,n)), n \geq 2, k \geq 2$connected with the vertices of the path P_2. Let $V(G) = \{v_0\} \cup \{v_1^1, v_2^1, v_3^1 \dots, v_{k-1}^1\} \cup \{v_1^2, v_2^2, v_3^2 \dots, v_{k-1}^2\} \cup \{v_1^3, v_2^3, v_3^3 \dots, v_{k-1}^3\} \cup \dots \cup \{v_1^n, v_2^n, v_3^n \dots, v_{k-1}^n\} \cup \{u_0\} \cup \{u_1^1, u_2^1, u_3^1 \dots, u_{k-1}^1\} \cup \{u_1^2, u_2^2, u_3^2 \dots, u_{k-1}^2\} \cup \{u_1^3, u_2^3, u_3^3 \dots, u_{k-1}^3\} \cup \dots \cup \{u_1^n, u_2^n, u_3^n \dots, u_{k-1}^n\} \cup \{w_0\} \cup \{w_1^1, w_2^1, w_3^1 \dots, w_{k-1}^1\} \cup \{w_1^2, w_2^2, w_3^2 \dots, w_{k-1}^2\} \cup \{w_1^3, w_2^3, w_3^3 \dots, w_{k-1}^3\} \cup \dots \cup \{w_1^n, w_2^n, w_3^n \dots, w_{k-1}^n\}$, $B(G) = \{B_1, B_2, B_3, \dots, B_{3n+1}\}$ denote the vertex set, edge set and the block set of three copies of windmill $Wd(k,n)), n \geq 2, k \geq 2$. $|V(G)| = 2(k-1)^n + 2$, $|B(G)| = 2n + 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first windmill $Wd(k,n)), n \geq 2, k \geq 2$. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of second windmill graph$Wd(k,n)), n \geq 2, k \geq 2$ respectively. Colors $c_i 1 \leq i \leq n$to the blocks $B_k$for $2n+1 \leq k \leq 3n$ of third windmill graph . Blocks B_{3n+1}&$B_{3n+2}$are colored with the colors which are not colored to the blocks B_1&B_{n+1}&B_{2n+1}. This block coloring is proper. The AUM block chromatic number for the three copies ofwindmill $Wd(k,n)), n \geq 2, k \geq 2$,joined by the path union of$P_3$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 12: The$n$ copies of friendship graph $F_3^n, n \geq 2$,joined by the path union ofP_n, the AUM block chromatic number of G is n.

Proof: Let $F_3^n, n \geq 2$,be the friendship graph. Considering n copies of friendship graphs F_3^nconnected with the n vertices of the path P_n.Let $V(G), E(G), B(G) = \{B_1, B_2, B_3, \dots, B_{n^2+n-1}\}$ denote the vertex set, edge set and the block set of *n* copies of friendship graph $F_3^n, n \geq 2$. $|V(G)| = n(2)^n + n + 1$, $|B(G)| = n^2 + n - 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first friendship graph. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of Second friendship graph respectively. Colors $c_i 1 \leq i \leq n$to the blocks $B_k$for $2n+1 \leq k \leq 3n$ of third friendship graph. The same colors are assigned to corresponding blocks of each copy of friendship graph.Blocks $B_{n^2+1}B_{n^2+2}$, B_{n^2+3},B_{n^2+4},...,B_{n^2+n-1} are colored with the color which are not colored to the blocks B_1&$B_{n+1}B_{2n+1}, B_{3n+1}, \ldots, B_{(n-1)n+1}$. This block coloring is proper. The AUM block chromatic number for the$n$ copies of friendship graph $F_3^n, n \geq 2$,joined by the path union of$P_n$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 13: The$n$ copies of of cactus graph $C_k^n, n \geq 2, k \geq 4$,joined by the path union ofP_n the AUM block chromatic number of Gis$n$.

Proof: Let $C_k^n, n \geq 2, k \geq 3$be the cactus graph. Considering n copies of cactus graph $C_k^n, n \geq 2, k \geq 4$connected with the n vertices of the path P_n. et $V(G), E(G), B(G) = \{B_1, B_2, B_3, \ldots, B_{n^2+n-1}\}$ denote the vertex set, edge set and the block set of n copies of cactus $C_k^n, n \geq 2, k \geq 4$. $|V(G)| = n(k-1)^n + 2$, $|B(G)| = n^2 + n - 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first cactus graph. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of Second cactus graph.Colors $c_i 1 \leq i \leq n$to the blocks $B_k$for $2n+1 \leq k \leq 3n$ of third cactus graph . The same colors are assigned to corresponding blocks of each copy of cactus graph.

Blocks $B_{n^2+1}B_{n^2+2}$, B_{n^2+3},B_{n^2+4},...,B_{n^2+n-1} are colored with the color which are not colored to the blocks B_1&$B_{n+1}B_{2n+1}, B_{3n+1}, \ldots, B_{(n-1)n+1}$. This block coloring is proper. The AUM block chromatic number for the$n$ copies of cactus graph $C_k^n, n \geq 2, k \geq 4$,joined by the path union of$P_n$is n. i.e., $\chi_{Bl}(G) = n$.

Theorem 14: The$n$ copies of of graph windmill graph $Wd(k,n)), n \geq 2, k \geq 2$,joined by the path union ofP_n, the AUM block chromatic number of Gis$n$.

Proof: Let $Wd(k,n)), n \geq 2, k \geq 2$be the windmill graph. Considering two windmill graph $Wd(k,n)), n \geq 2, k \geq 2$connected with the n vertices of the path P_n. Let $V(G)$, E(G), $B(G) = \{B_1, B_2, B_3, \ldots, B_{3n+1}\}$ denote the vertex set, edge set and the block set of n copies of windmill $Wd(k,n)), n \geq 2, k \geq 2$. $|V(G)| = 2(k-1)^n + 2$, $|B(G)| = 2n + 1$.

Assign the colors $c_i$to the blocks $B_i$for $1 \leq i \leq n$ of first windmill $Wd(k,n)), n \geq 2, k \geq 2$. Colors $c_i 1 \leq i \leq n$to the blocks $B_j$for $n+1 \leq j \leq 2n$ of second windmill graph$Wd(k,n)), n \geq 2, k \geq 2$ respectively. Colors $c_i 1 \leq i \leq n$to the blocks $B_k$for $2n + 1 \leq k \leq 3n$ of third windmill graph . The same colors are assigned to corresponding blocks of each copy of windmill graph.

Blocks $B_{n^2+1}B_{n^2+2}$, B_{n^2+3},B_{n^2+4},...,B_{n^2+n-1} are colored with the color which are not colored to the blocks B_1&$B_{n+1}B_{2n+1}, B_{3n+1}, \ldots, B_{(n-1)n+1}$. . This block coloring is proper. The AUM block chromatic number for the$n$ copies ofwindmill $Wd(k,n)), n \geq 2, k \geq 2$,joined by the path union of$P_n$is n. i.e., $\chi_{Bl}(G) = n$.

4. CONCLUSION

In this paper, we have introduced the new definition of block coloring, and found that the block chromatic number ofpath, cycle, complete graph,n −barbell graph, windmill graph, friendship graph, cactus graph, bipartite graph and their extensions. The work has huge scope for further continuation and application in power industry and management etc.

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