A NOTE ON sg* CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce sg*closed set in a Soft topological space and to study some of its properties. Then sg* continuous mapping and irresolute mapping areintroduced and some of its properties are studied. The concept *sg open, sg* closed mappings and *sg**homeomorphism are introduced and their properties are studied.**

Key-Words: sg* continuous mapping , irresolute mapping, *sg homeomorphism**

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1 . INTRODUCTION

The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. In the year 1999, Molodtsov[1] initiated the study of soft sets. Soft set theory has been applied in several directions. Following this Maji, Biswas, and Roy[7,8] discussed soft set theoretical operations and gave an application of soft set theory to a decision making problem. Recently Muhammad Shabir and Munazza Naz introduced the notion of soft topology[10] and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. Several mathematicians published papers on applications of soft sets and soft topology[1,2,6,11,12,1]. Soft sets and soft topology have applications to data mining, image processing, decision making problems, spatial modeling and neural patterns[3,4,5,7]. In this paper, the concept sg*closed set is introduced in soft topological space and the concept of sg* continuous mapping and *sg** irresolute mapping are introduced and some of their properties are studied. Further the concept *sg** open , sg* closed mappings and *sg**homeomorphism are introduced and some of their basic soft topological properties are investigated. Finally the concept of slightly *sg** continuous mapping is introduced and studied some of its basic concepts.

2. PRELIMINARIES

2.1 Definition A soft set *(A,E)* is called sg* closed in a soft topological space $(X, \tilde{\tau}\, E)$ of $cl\,(A,E) \tilde{\subseteq} (U,E)$whenever $(A,E) \tilde{\subseteq} (U,E)$ and $(U,E)$is soft g open in $\tilde{X}$.

2.2.1 Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$and

$\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (A_1,E), (A_2,E), (A_3,E), (A_4,E), (A_5,E), (A_6,E), (A_7,E)\}$ where

$(A_1,E) = \{(b_1,\{a_2\}), (b_2,\{a_1\}),$ $(A_2,E) = \{(b_1,\{a_2\}), (b_2,X)\}$

$(A_3,E) = \{(b_1,\{a_{2,}a_3\}), (b_2,\{a_{2,}a_3\})\},$ $(A_4,E) = \{(b_1,\{a_{1,}a_3\}), (b_2,X)\},$

$(A_5,E) = \{(b_1,\emptyset)\{b_{2,}\{a_1\}\}$ $(A_6,E) = \{(b_1,\emptyset)\{b_{2,}\{a_{2,}a_3\}\}$ and

$(A_7,E) = \{(b_1,\emptyset), (b_2,X)\}.$

Clearly $(A,E) == \{(b_1,\{a_{1,}a_3\})(b_2,\{a_3\})\}$ is sg* closed in $(X, \tilde{\tau}\, E)$.

since for *(A,E)* there exists a soft g open set $(U,E) = \{(b_1,\{a_{1,}a_3\}.(b_{2,}\{a_{2,}a_3\})$ such that $cl\,(A,E) \tilde{\subseteq} (U,E)$.

2.1 Theorem

Every soft closed set is sg* closed in a soft topological space $(X, \tilde{\tau}\, E)$.

3. sg* CONTINUOUS MAPPINGS

3.1 Definition

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* continuous if $f^1\,(U,E)$ is sg* closed in $(X, \tilde{\tau}, E)$ for every soft closed set *(U,E)* of $(X, \tilde{\omega}, E)$.

3.2. Theorem

Let $f: \tilde{X} \longrightarrow \tilde{Y}$ be a soft mapping from soft topological space $(X, \tilde{\tau}, E)$ into a soft topological space $(X, \tilde{\tau}, E)$. Then the following statements are equivalent.

i) $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* continuous.

ii) The inverse image of each soft open set in $\tilde{Y}$ is sg* open in $\tilde{Y}$.

iii) For each soft subset $(A,E) \tilde{\in} (Y, \tilde{\omega}, E) sg^*cl(f^{-1}\,(A,E)) \tilde{\subseteq} f^{-1}\,cl(A,E))$.

i v) For each soft subset $(B,E) \tilde{\in} (X,\tilde{\tau}, E) f\left(sg^{*}cl\,(B,E)\right) \tilde{\subseteq} cl\left(f(B,E)\right)$.

Proof (i) ⟶ (ii) follows from 3.1 Definition.

(i)⟶(iii)

Let *(A,E)* be a soft subset of $(Y,\tilde{\omega},E)$. By 3.2.1 Definition $f^{-1}\,cl(A,E))$ is a sg* closed set containing $f^{-1}\,(A,E)$ and $sg^{*}cl\,(f^{-1}\,(A,E)) \tilde{\subseteq} f^{-1}\,cl(A,E))$.

(iii)⟶(iv)

Let $(B,E) \tilde{\in} (Y,\tilde{\tau},E)$, then $f(B,E) \tilde{\in} (Y,\tilde{\omega},E)$ Hence from (iii) $sg^{*}cl\,(f^{-1}\left(f(B,E)\right) \tilde{\subseteq} f^{-1}(\,cl(A,E))$. Therefore $f\left(sg^{*}cl(B,E)\right) \tilde{\subseteq} clf\,(B,E)$.

(iv)⟶(i)

Let *(U,E)* be a soft closed set in $\tilde{Y}$. Then by (iv)

$f\left(sg^{*}cl\left(f^{-1}(U,E)\right)\right) \tilde{\subseteq} cl(f(f^{-1}(U,E))$. Hence $sg^{*}cl\,(f^{-1}(U,E) \tilde{\subseteq} f^{-1}(U,E)$. Therefore $f^{-1}(U,E)$ is a sg* closed set in $\tilde{X}$.

3.3 Theorem

Let $f\colon \tilde{X} \longrightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg* continuous.

Proof

(i) ⟶ (ii) follows from 3.1 Definition.

(i)⟶(iii)

Let *(A,E)* be a soft subset of $(Y,\tilde{\omega},E)$. By 3.1 Definition $f^{-1}(cl\,(A,E)$ is a sg* closed set containing $f^{-1}(A,E)$ and $sg^{*}cl\,(f^{-1}(A,E)) \tilde{\subseteq} f^{-,\{1}(cl\,(A,E))$.

(iii)⟶(iv)

Let $(B,E) \tilde{\subseteq} (X,\tilde{\tau},E)$.Then $f(B,E) \tilde{\in} (Y,\tilde{\omega},E)$. Hence from (iii) $sg^{*}cl\,(f^{-1}(f(B,E)))$ $\tilde{\subseteq} f^{-1}\left(clf(B,E)\right)$. Therefore $f\left(sg^{*}cl\,(B,E)\right) \tilde{\subseteq} clf\,(B,E)$.

(iv)⟶(i)

Let *(U,E)* be a soft closed set in $\tilde{Y}$. Then by (iv)

$f(sg^{*}cl(f^{-1}(U,E))) \tilde{\subseteq} cl(f(f^{-1}(U,E)))$. Hence $sg^{*}cl(f^{-1}(U,E) \tilde{\subseteq} f(U,E)$.

Therefore $f^{-1}(U,E)$ is a sg* closed set in $\tilde{X}$.

3.4 Theorem

Let $f: \tilde{X} \longrightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg* continuous.

Proof

Let *(A,E)* be any soft closed set in $\tilde{Y}$. Then $f^{-1}(A,E)$ is soft closed in $\tilde{X}$. Therefore by 2.1 Theorem, $f^{-1}(A,E)$ is sg* closed in $\tilde{X}$.

3.5 Example

The following example shows that the converse of the above 3.2.2 Theorem need not be true.

Let $X = \{a_1, a_2, a_3\}, Y = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{\tau_1} = \{\tilde{\emptyset}, \tilde{X}, (B_1,E), (B_2,E), (B_3,E), (B_4,E), (B_5,E)\}$

$\tilde{\tau_1} = \{\tilde{\emptyset}, \tilde{X}, (A_1,E), (A_2,E), (A_3,E), (A_4,E), (A_5,E), (A_6,E)\}$ be two soft topological spaces over X and Y respectively. Then $(B_1,E), (B_2,E), (B_3,E), (B_4,E), (B_5,E)$ are soft sets over X and $(A_1,E), (A_2,E), (A_3,E), (A_4,E), (A_5,E)$ are soft sets over Y defined as follows:

$(A_1,E) = \{(b_1,\{a_2,a_3\}), (b_2,\{a_1,a_3\})\}$, $(A_2,E) = \{(b_1,\{a_3\}), (b_2,\{a_1\})\}$,

$(A_3,E) = \{(b_1,\{a_2\}), (b_2,\{a_3\})\}$, $(A_4,E) = \{(b_1,\{a_3\}), (b_2,\emptyset)\}$,

$(A_5,E) = \{(b_1,X), (b_2,\{a_1,a_3\})\}$, $(A_6,E) = \{(b_1,\{a_{2,}a_3\}), (b_2,\{a_3\})\}$,

$(B_1,E) = \{(b_1,\{a_2\}), (b_2,\{a_1\})\}$ $(B_2,E) = \{(b_1,\{a_3\}), (b_2,\{a_1,a_3\})\}$,

$(B_3,E) = \{(b_1,\{a_2,a_3\}), (b_2,\{a_1,a_2\})\}$, $(B_4,E) = \{(b_1,X), (b_2,\{a_1,a_2\})\}$,

and $(B_5,E) = \{(b_1,\emptyset), (b_2,\{a_1\})\}$.

Let $f: \tilde{X} \longrightarrow \tilde{Y}$ be a soft mapping defined by $f(a_1) = a_1$, $f(a_2) = a_3$, and $f(a_3) = a_2$. Then f is sg* continuous map but not soft continuous. Since $f^{-1}(A_1,E) = \{(b_1,\{a_{2,}a_3\}), (b_2,\{a_1,a_{2,}\})\}$,

$f^{-1}(A_2,E) = \{(b_1,\{a_2\}), (b_2,\{a_1\})\}$, $f^{-1}(A_3,E) = \{(b_1,\{a_3\}), (b_2,\{a_2\})\}$,

$f^{-1}(A_4,E)=\{(b_1,\{a_2\}),(b_2,\emptyset)\}$, $f^{-1}(A_5,E)=\{(b_1,X),(b_2,\{a_1,a_{2,}\})\}$,

$f^{-1}(A_6,E)=\{(b_1,\{a_2,a_3\}),(b_2,\{a_{2,}\})$ are sg* open sets in $\widetilde{\tau_1}$ but

$f^{-1}(A_3,E)$, $f^{-1}(A_4,E), f^{-1}(A_5,E), f^{-1}(A_6,E)$ are not soft open sets in $\widetilde{\tau_1}$.

3.6 Theorem

If $f:\tilde{X} \longrightarrow \tilde{Y}$ is a sg* continuous mapping from $\tilde{X}$ into $\tilde{Y}$ then f is soft g continuous.

Proof Let (*A,E*) be any soft closed set in $\tilde{Y}$. Then f^{-1} (A,E) is sg* closed in $\tilde{X}$. Therefore by 2.1 Theorem f^{-1} $(A,E)$is soft g closed in $\tilde{X}$.

3.7 Definition

A soft mapping $f:\tilde{X} \longrightarrow \tilde{Y}$ called sg* irresolute if $f^{-1}(U,E)$ is sg* closed in $\tilde{X}$ for every sg* closed set of $(Y,\widetilde{\omega},E)$.

3.8 Remark

A soft mapping $f:\tilde{X} \longrightarrow \tilde{Y}$ is sg* irresolute if and only if the inverse image of every sg* open set in $(Y,\widetilde{\omega},E)$ is sg* open in $\tilde{X}$.

3.9 Theorem If $f:\tilde{X} \longrightarrow \tilde{Y}$ and $h:\tilde{Y} \longrightarrow \tilde{Z}$ are any two soft mappings then

i) $h\circ g$ is sg* continuous if h is soft continuous and f is sg* continuous.

ii) $h\circ g$ is sg* continuous if h is sg* continuous and g is sg* irresolute.

iii) $h\circ g$ is sg* irresolute if both g and h are sg* irresolute.

Proof

(i) Let *(U,E)* be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U,E)$ is soft closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U,E)) = h^o\ g)(U,E)$ is sg* closed in $\tilde{X}$.

(ii) Let *(U,E*) be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U,E)$ is sg* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U,E)) = h^o\ g)(U,E)$ is sg* closed in $\tilde{X}$.

(iii) Let (U,E) be a sg* closed set in $\tilde{Z}$. Then $h^{-1}(U,E)$ is sg* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U,E)) = h^o\ g)(U,E)$ is sg* closed in $\tilde{X}$.

3.10 Theorem

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* irresolute if and only if for every soft subset *(U,E)* of $\tilde{X}, g\big(sg^* \, cl(U,E)\big) \tilde{\subseteq} sg^* \, cl\big(g(U,E)\big)$.

Proof Let g be a sg* irresolute mapping and *(U,E)* be a soft subset in $\tilde{X}$. Then $sg^* \, cl\big(g(U,E)\big)$ is sg* closed set in $\tilde{Y}$. Hence $g^{-1}(sg^* \, cl\big(g(U,E)\big))$ is sg* closed in $\tilde{X}$ and $(U,E) \tilde{\subseteq} g^{-1}\big(g(U,E)\big) \tilde{\subseteq} g^{-1}(sg^* \, cl\big(g(U,E)\big))$.

Therefore

$sg^* \, cl(U,E) \tilde{\subseteq} g^{-1}(sg^* \, cl\big(g(U,E)\big))$, hence $g(sg^* \, cl(U,E)) \tilde{\subseteq} g^{-1}(sg^* \, cl\big(g(U,E)\big)$.

Conversely, suppose that *(U,E)* is sg* closed in $\tilde{Y}$.

Therefore

$g(sg^* \, cl\big(g^{-1}(U,E)\big)) \tilde{\subseteq} (sg^* \, cl\left(g\big(g^{-1}(U,E)\big)\right) = sg^* \, cl(U,E) = (U,E)$. Hence $sg^* \, cl\big(g^{-1}(U,E)\big) \tilde{\subseteq} g^{-1}(U,E)$.

4. sg* HOMEOMORPHISMS

4.1 Definition

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* open if $g(U,E)$ of each soft open set *(U,E)* in

$(X, \tilde{\tau}, E)$ is sg* open in $(Y, \widetilde{\omega}, E)$.

4.2 Definition

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* closed if $g(U,E)$ of each soft closed set *(U,E)* in $(X, \tilde{\tau}, E)$ is sg* closed in $(Y, \widetilde{\omega}, E)$.

4.3 Theorem

Let the soft mappings $f: \tilde{X} \longrightarrow \tilde{Y}$ and $g: \tilde{Y} \longrightarrow \tilde{Z}$ be bijective. If $g \circ f: \tilde{X} \longrightarrow \tilde{Z}$ is soft continuous and $f: \tilde{X} \longrightarrow \tilde{Y}$ is soft continuous and $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* closed then $g: \tilde{Y} \longrightarrow \tilde{Z}$ is sg* continuous.

Proof

Let *(U,E)* be the soft closed set in $\tilde{Z}$. Since $g \circ f: \tilde{X} \longrightarrow \tilde{Z}$ is soft continuous, then $f^{-1}\left(g^{-1}(U,E)\right) = (g \circ f)^{-1}(U,E)$ is soft closed set in $\tilde{X}$. Since $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* closed, then $f\left(f^{-1}\left(g^{-1}(U,E)\right)\right) = g^{-1}(U,E)$ is sg* closed set in $\tilde{Y}$.

4.5 Theorem

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is a sg* open iff if $f\left(int(B,U)\right) \tilde{\subseteq} sg^{*}int\ (f(B,E))$ for every soft subset *(B,E)* of $\tilde{X}$.

Proof

Let $f: \tilde{X} \longrightarrow \tilde{Y}$ be sg* open and *(B,E)* be a soft subset of $\tilde{X}$, then $int(B,U)$ is a soft open set in $\tilde{X}$. Hence $f\left(int(B,E)\right) = sg^{*}int\ (f(int(B,E))$.

Conversely, Let *(G,E)* be a soft open set in $\tilde{X}$. $f(G,E) = f\left(int(G,E)\right) \tilde{\subseteq} sg^{*}int\ (f(G,E))$, which implies $f(G,E) \tilde{\subseteq} sg^{*}int\ (f(G,E))$. Hence $f(G,E)$ is a sg* open in $\tilde{Y}$.

4.6 Definition

If a soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* continuous bijective and f^{-1} is sg* continuous then *f* is said to be sg* homeomorphism from $(X, \tilde{\tau}, E)$ in to $(Y, \tilde{\omega}, E)$.

4.7 Theorem

Let $f: \tilde{X} \longrightarrow \tilde{Y}$ be the soft bijective mapping. Then the following statements are equivalent: . Since f is sg* open map,

i) $f^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous.

ii) f is sg* open .

iii) f is sg* closed.

Proof

(i)⟶(ii) Let *(U,E)* be any soft open set in $\tilde{X}$. Since $f^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous, therefore $(f^{-1})^{-1}(U,E) = f(U,E)$ is sg* open in $\tilde{Y}$.

(ii)⟶(iii) Let *(B,E)* be any soft closed set in $\tilde{X}$, then $\tilde{X}-(B,E)$ is soft open set in $\tilde{X}$. Since f is sg* open map, $f(\tilde{X}-(B,E))$ is sg* open in $\tilde{Y}$. But $f\left(\tilde{X}-(B,E)\right)=\tilde{Y}-f(B,E)$, implies $f(B,E)$is sg* closed in $\tilde{Y}$.

(iii)⟶(i) Let *(B,E)* be any soft closed set in $\tilde{X}$. Then $(f^{-1})^{-1}(U,E)=f(U,E)$ is sg* closed in $\tilde{Y}$. Therefore $f^{-1}\colon\tilde{Y}\ \longrightarrow\ \tilde{X}$ is sg* continuous.

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