Power Dominator Coloring for Vertex Switching of some Graphs

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***Abstract* — Let G = (V, E) be a finite, and undirected graph without loops and multiple edges. Vertex switching was one of the most important vertex operation studied in graph theory. Vertex switching will give new dimension in the field of power dominator coloring, since this vertex operation ensures the connection between vertices. We find the power dominator coloring for the graphs path,cycle, double fan graph, with the context of vertex switching.**

Keywords — Coloring, Power dominator coloring, switching, double fan graph, path, cycle.

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Introduction

Francis Guthrie, in 1852 used graph coloring to color the counties in the map of England and found only four colors were required to color all countries such that, no two countries that have a common border was colored the same color, which in turn initiated the theory of graph coloringand first proved by A.B. Kempe in[1]. In the year 1960, graph domination was introduced [[2]]. About 1222 papers on 75 types of the domination were listed in the article by Teresa W. et. al. which was published in 1998 [3].

By combining graph coloring and graphdomination, dominatorcoloring was introduced by Gera[4]. There are several variants of dominator coloring were studied recently namely global coloring [5], total dominator coloring[6] and rainbow dominator coloring[7].

Power domination was introduced in early 1990 when Haynes et al [8]trying to place minimum number of Phase Measurement Units to observe current flow in the circuit. K. S. Kumar, N. G. David, and K. G. Subramanian in 2016, introduced a new dominator coloring by combining powerdomination and coloring, namely, powerdominatorcoloring[9]. A. Uma Maheswari and Bala Samuvel J, studied power dominator chromatic number for some special graphs [[10], [11],[12].

A. Edward Samuel and S. Kalaivanistudied more about Prime labellingfor vertex switching operation of Fan related graphs in [13], [14] Here, we find the power dominator chromatic number for the graphs path, cycle, complete graph, bipartite graph, double fan graph, with the context of vertex switching.

I. PRELIMINARIES

The definitions required for this paper are recalled below from [15].

Definition 1: Dominator Coloring[12]

A dominator coloring [[16]–[18]] of a graph is a proper coloring such that each vertex dominates every vertex in at least one color class consisting of vertices with the same color. The chromatic number $\chi_d(G)$ of a graph is the minimum number of colors needed in a dominator coloring of G.

Definition 2: Monitoring Set [15]

For a vertex $v \in G$, a monitoring set $M(v)$[15] is associated as follows:
Step(i): M(v) = N[v], the closed neighborhood on v.
Step(ii): Add a vertex u to M(v), (which is not in M(v)) whenever u has a neighbor w ϵ M(v) such that all the neighbors of w other than u, are already in M(v).
Step(iii): Repeat step(ii) if no other vertex could be added to M(v).
Then we say that *v* power dominates the vertices in *M*(*v*).
Note that if a vertex *v* dominates [19] another vertex *u,* then *v* power dominates *u* but the converse need not be true.

Definition : Power Dominator Coloring[7]

The power dominator coloring [[9],[10]] of G is a proper coloring of G, such that every single vertex of G power dominates all vertices of some color class. The minimum number of color classes in a power dominator coloring of the graph, is the power dominator chromatic number. It is denoted by $\chi_{pd}(G)$.

Definition 4: Switching a Vertex [[13]

A vertex switching of a graph is obtained by taking a vertex v of G, removing all the entire edges incidentwith $v$and adding edges joining v to every vertex which are not adjacent to v in G.

II. MAIN RESULTS

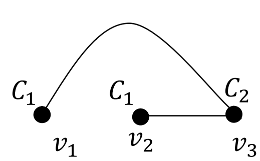
Theorem 1

For $n = 3$,the power dominator chromatic number for the modified graph P_n'obtained by switching any pendent vertex in path P_n is 2.

Proof: Let P_3 be the path with 3 nodes. Let v_1, v_2, v_3be the verticesof Path P_3, where $v_1$and v_3 are the pendent vertices. Let $E(P_3) = \{v_1v_2, v_2v_3\}$.

We switch any pendent vertex $\{v_1\}$or $\{v_3\}$. Without loss of generality we consider pendent vertex v_3 is switched, such that graph P_3' is obtained with vertices $\{v_1, v_2, v_3\}$ and edges $E(P_n') = \{v_1v_3, v_2v_3\}$.

In figure 1, the power dominator coloring of modified graphP_3'obtained by a switching vertex v_1 in path $P_3$is shown:



Assign the colorc_1 to vertices v_1, v_2. Color c_2 be assigned to the odd indexed vertex v_3.This coloring is proper. Every vertex $\{v_1, v_2, v_3\}$ of graph P_3' will power dominate all the vertices of the graph P_3'. And vertex v_3 will power dominate itself and color class c_1. Therefore, every vertex in the graph P_3' will power dominate atleast one color class. The power dominator chromatic number for the graph P_3'obtained by switching a pendent vertex v in path P_3 is 2. i.e., $\chi_{pd}(P_3') = 2$. □

Theorem 2

For $n = 3$,the power dominator chromatic number for the modified graph P_n'obtained by switching vertex v_2 in path P_n is 3.

Proof: Let P_3be the path with 3 nodes. Let v_1, v_2, $v_3$be the vertices of path P_3, where $v_1$and v_3 are the pendent vertices. Let $E(P_3) = \{v_1v_2, v_2v_3\}$. Now, vertex $v_2$is switched, such that graph$P_3'$is obtained with vertices $\{v_1, v_2, v_3\}$ and with no edges.

Assign the colorc_1 to vertices v_1, Color c_2 be assigned to the even indexed vertex v_2 and Color c_3 be assigned to the vertex v_3.Thiscoloring is proper. Every vertex $\{ v_1, v_2, v_3\}$ of graph P_3' will power dominate themselves by the definition.Therefore, every vertex in the graph P_3' will power dominate one color class. The power dominator chromatic number for the graph P_3'obtained by switching a pendent vertex v in path P_3 is 3. i.e., $\chi_{pd}(P_3') = 3$. □

Remark: When $n = 2$, The power dominator chromatic number for the modified graph P_n'obtained by switching any vertex in path P_n is 2.

Theorem 3

For $n \geq 4$,the power dominator chromatic number for the modified graph P_n'obtained by switching any pendent vertex in path $P_n \chi_{pd}(P'_n) = \begin{cases} 3, if\ n = 4 \\ 4, n \geq 4 \end{cases}$

Proof: Based on the number of vertices in the path P_n, we have following cases.

Case (i) Let $n = 4$

Let P_4 be the path with 4 nodes. Let the vertices be v_1, v_2, v_3, v_4, where $v_1$and v_4 are the pendent vertices. Let $\mathrm{E}(P_n) = \{v_1v_2, v_2v_3, v_3v_4\}$.

We switch any pendent vertex $\{v_1\}$or $\{v_4\}$. Without loss of generality we consider pendent vertex v_1 is switched, such that graph P_4' is obtained with vertices $\{v_1, v_2, v_3, v_4\}$ and edges $E(P_4') = \{v_1v_3, v_1v_4, v_2v_3, v_3v_4\}$.

Assign the colorc_1 to vertices v_1. Color c_2 be assigned to the even indexed vertices $v_2, \& v_4$. Color c_3 be assigned to vertex v_3. This coloring is proper. Then every vertex in$\{v_1, v_2, v_3, v_4\}$ of graph P_4' will power dominate the vertex v_3 of the graph P_4'. Vertex v_3 will power dominate itself and color class c_1. Therefore, every vertex inthe graph P_4' will power dominate atleast one color class. The power dominator chromatic number for the graph P_4'obtained by switching a pendent vertex v in path P_4 is 3. i.e., $\chi_{pd}(P_4') = 3$.

Case (ii) Let $n \geq 5$

Let P_n be the path with n nodes. Let $v_1, v_2, v_3, \dots, v_n$be the vertices of path P_n, where $v_1$and v_n are the pendent vertices. Let $E(P_n) = \{v_1v_2, v_2v_3, \dots, v_{n-2}v_{n-1}, v_{n-1}v_n\}$.

We switch any pendent vertex $\{v_1\}$ or $\{v_n\}$. Without loss of generality we consider pendent vertex v_1 is switched, to attain graph P_n' with vertices $\{v_1, v_2, v_3, \dots, v_n\}$ and edges $E(P_n') = \{v_1v_3, v_1v_4, \dots, v_1v_n, v_2v_3, v_3v_4, \dots, v_{n-1}v_n\}$.

Assign the colorc_1 and color c_2 to vertices $v_1$and v_2 respectively. Color c_3 be assigned to even indexed vertices $v_{2l}, 2 \le l \le \left\lfloor\frac{n}{2}\right\rfloor$. Color c_3 be assigned odd indexed vertices $v_{2l+1}, 2 \le l \le \left\lfloor\frac{n}{2}\right\rfloor$. This coloring is proper. Then vertices $\{v_1, v_3, v_4, \dots, v_n\}$ of graph P_n' will power dominate the vertex v_1 of the graph P_n'. Vertex v_2 will power dominate itself. Therefore, every vertex in the graph P_n' will power dominate atleast one color class. The power dominator chromatic number for the graph P_n'obtained by switching a pendent vertex v in path P_n is 4. i.e., $\chi_{pd}(P_n') = 4$. □

Theorem 4

For $n \ge 4$,the power dominator chromatic number for the modified graph P_n'obtained by switching any support vertex in path P_n $\chi_{pd}(P'_n) = \begin{cases} 3, if\ n = 4 \\ 4, if\ n = 5 \\ 5, if\ n \ge 6 \end{cases}$

Proof: Based on the number of vertices in the path P_n, we have following cases.

Case (i) Let $n = 4$

Let P_4 be the path with 4 nodes. Let the vertices be v_1, v_2, v_3, v_4 where $v_2$and v_3 are the support vertices. Let $E(P_4) = \{v_1v_2, v_2v_3, v_3v_4\}$.

We switch any support vertex $\{v_2\}$or $\{v_3\}$. Without loss of generality we consider support vertex v_2 is switched, such that graph P_4' is obtained with vertices $\{v_1, v_2, v_3, v_4\}$ and edges $E(P_4') = \{v_2v_3, v_3v_4\}$.

Assign the colorc_1 to vertex v_1. Color c_2 can be assigned to the vertices $v_2 \& v_3$. Color c_3 can be assigned to vertex v_4. This coloring is proper. Every vertex $\{v_2, v_3, v_4\}$ of graph P_4' will power dominate the color class c_2 and color class c_3 of the graph P_4'. Vertex v_1 will power dominate itself. Therefore, every vertex in the graph P_4' will power dominate atleast one color class. The power dominator chromatic number for the graph P_4'obtained by switching a pendent vertex v in path P_4 is 3. i.e., $\chi_{pd}(P_4') = 3$.

Case (ii) Let $n = 5$

Let P_5 be the path with 5 nodes. Let the vertices be v_1, v_2, v_3, v_4, v_5 where $v_2$and v_4 are the support vertices. Let $E(P_5) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$.

We switch any support vertex $\{v_2\}$or $\{v_4\}$. Without loss of generality we consider support vertex v_2 is switched, such that graph P_5' is obtained with vertices $\{v_1, v_2, v_3, v_4, v_5\}$ and edges $E(P_5') = \{v_2v_4, v_2v_5, v_3v_4, v_4v_5\}$.

Assign the colorc_1 to vertex v_1. Color c_2 be assigned to the vertex v_2. Color c_3 be assigned to vertices $v_3 \& v_5$, and vertex v_4 is colored with color c_4.This coloring is proper. Every vertex

$\{v_2, v_3, v_5\}$ of graph P_5' will power dominate the color class c_4. Vertex v_1, v_2 and v_4 will power dominate themself. Therefore, every vertex in the graph P_5' will power dominate atleast one color class. The power dominator chromatic number for the graph P_5'obtained by switching a pendent vertex v in path P_5 is 3. i.e., $\chi_{pd}(P_5') = 3$.

Case (iii) Let $n \geq 6$

Let P_n be the path with n nodes. Let $v_1, v_2, v_3, \ldots, v_n$be the vertices of path P_n, where v_2 and v_{n-1} are any support vertices. Let $E(P_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-2}v_{n-1}, v_{n-1}v_n\}$.

We switch any support vertex $\{v_2\}$ or $\{v_{n-1}\}$. Without loss of generality we consider support vertex v_2 is switched, to attain graph P_n' with vertices $\{v_1, v_2, v_3, \ldots, v_n\}$.

Assign the colorsc_1, c_2&c_3 to vertices v_1, v_2&v_3 respectively. Color c_4 be assigned to even indexed vertices $v_{2l}, 2 \leq l \leq \left\lfloor \frac{n}{2} \right\rfloor$. Color c_5 be assigned odd indexed vertices $v_{2l+1}, 2 \leq l \leq \left\lfloor \frac{n}{2} \right\rfloor$. This coloring is proper. Vertices $\{v_4, \ldots, v_n\}$ of graph P_n' will power dominate the vertex v_2 of the graph P_n'. Vertex v_1, v_2, v_3 will power dominate themselves. Therefore, every vertex in the graph P_n' will power dominate atleast one color class. The power dominator chromatic number for the graph P_n'obtained by switching a pendent vertex v in path P_n is 5. i.e., $\chi_{pd}(P_n') = 5$. □

Theorem 5

For $n \geq 5$,the power dominator chromatic number for the modified graph P_n'obtained by switching either vertex v_3 or vertex v_{n-2} in Path $P_n \chi_{pd}(P'_n) = \begin{cases} 2, if n = 5 \\ 4, if\ n \geq 6 \end{cases}$

Proof: Based on the number of vertices in the path P_n, we have following cases.

Case (i) Let $n = 5$

Let P_5 be the path with 5 nodes. Let the vertices be v_1, v_2, v_3, v_4, v_5. Let $E(P_5) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$.We switch a vertex $v_3$such that graph P_5' is obtained with vertices $\{v_1, v_2, v_3, v_4, v_5\}$ and edges $E(P_5') = \{v_1v_2, v_1v_3, v_3v_5, v_4v_5\}$.

Assign the colorc_1 to vertex v_1, v_3, and v_4. Color c_2 be assigned to the vertices $v_1$andv_5. This coloring is proper. Every vertex $\{v_1, v_2, v_3, v_4, v_5\}$ of graph P_5' will power dominate the color class $c_1$and color classc_2.Therefore, every vertex in the graph P_5' will power dominate atleast one color class. The power dominator chromatic number for the graph P_5'obtained by switching a pendent vertex v in path P_5 is 3. i.e., $\chi_{pd}(P_5') = 3$.

Case (ii) Let $n \geq 6$

Let P_n be the path with n nodes. Let $v_1, v_2, v_3, \ldots, v_n$be the vertices of path P_n, where v_2 and v_{n-1} are any support vertices. Let $E(P_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-2}v_{n-1}, v_{n-1}v_n\}$.We switch either vertex $\{v_3\}$ or vertex $\{v_{n-2}\}$. Without loss of generality we consider vertex v_3 is switched, to attain graph P_n' with vertices $\{v_1, v_2, v_3, \ldots, v_n\}$.

Assign the colorc_1 to vertices $\{v_1\}$ & $\left\{v_{2l+1,}, 2 \leq l \leq \left\lfloor \frac{n}{2} \right\rfloor\right\}$. Color c_1 be assigned to even indexed vertices $\{v_2\}$ & $\left\{v_{2l}, 3 \leq l \leq \left\lfloor \frac{n}{2} \right\rfloor\right\}$. Color c_3 be assigned to vertex v_3 and color c_4 be assigned to vertex v_4.

This coloring is proper. Vertices $\{v_1, v_2, v_5, v_6, v_7, \ldots, v_n\}$ of graph P_n' will power dominate the vertex v_3. Vertices v_3, v_4 will power dominate themselves. Therefore, every vertex in the graph P_n' will power dominate atleast one color class. The power dominator chromatic number for the graph P_n'obtained by switching a pendent vertex v in path P_n is 4. i.e., □ $\chi_{pd}(P_n') = 4$.

Theorem 6

For $n \geq 7$,the power dominator chromatic number for the modified graph P_n'obtained by switching a vertex which is neither pendentnor support and neither the vertex v_3 nor the vertex v_{n-2} in path P_n,$\chi_{pd}(P'_n) = 5$.

Proof: Based on the number of vertices in the path P_n, we have following cases.

Case (i) Let $n = 7$
Let P_7 be the path with 7 nodes. Let the vertices be $v_1, v_2, v_3, v_4, v_5, v_6, v_7$. Let $E(P_7) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7\}$.

We switch avertex $v_4$which is neither a non-pendent, nor a non-support vertex,such that graph P_7' is obtained with vertices $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and edges $E(P_7') = \{v_1v_2, v_1v_3, v_3v_5, v_4v_5, v_6v_7\}$.Assign the color$c_1$ to vertices $\{v_1, v_7\}$. Color c_2 to vertices $\{v_2, v_6\}$. Colors c_3, c_4, c_5 be assigned to the vertices v_3, v_4andv_5.

This coloring is proper. Every vertex $\{v_1, v_2, v_6, v_7\}$ of graph P_5' will power dominate the color class c_4. And vertices v_3, v_4, v_5 will power dominate themselves. Therefore, every vertex in the graph P_7' will power dominate atleast one color class. The power dominator chromatic number for the graph P_7'obtained by switching a vertex $v_4$which is neither a non-pendent, nor a non-support vertex in path P_7 is 5. i.e., $\chi_{pd}(P_7') = 5$.

Case (ii) Let $n \geq 8$

Let P_n be the path with n nodes. Let $v_1, v_2, v_3, \ldots, v_n$be the vertices of path P_n, where v_2 and v_{n-1} are any support vertices. Let $E(P_n) = \{v_1v_2, v_2v_3, \ldots, v_{n-2}v_{n-1}, v_{n-1}v_n\}$.

We switch a vertex which is neither a non-pendent, nor a non-support and neither vertex v_3 nor vertex v_{n-2}. Without loss of generality, we consider vertex v_4 is switched, to attain graph P_n' with vertices $\{v_1, v_2, v_3, \ldots, v_n\}$.

Assign the colorc_1 to vertices $\{v_1\}$ & $\left\{v_{2l+1,}, 3 \leq l \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$. Color c_2 to vertices $\{v_2\}$ & $\left\{v_{2l,}, 3 \leq l \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$. Colors c_3, c_4, c_5 be assigned to the vertices v_3, v_4andv_5.This coloring is proper. Then vertices $\{v_1, v_2, v_6, v_7, v_8, \ldots, v_n\}$ of graph P_n' will power dominate the vertex v_4. And vertices $v_3, v_4$and v_5 will power dominate themselves. Therefore, every vertex in the graph P_n' will power dominate atleast one color class. The power dominator chromatic number for the graph P_n'obtained by switching a pendent vertex v in path P_n is 5. i.e., $\chi_{pd}(P_n') = 5$.

□

Theorem 7:

For any $3 \leq n \leq 6$,the power dominator chromatic numberfor the modified graph C'_nobtained by a switching any arbitrary vertex in cycle $C_n = \begin{cases} 3, if\ n = 3,5 \\ 2, if\ n = 4 \\ 4, if\ n = 6 \\ 5, if\ n \geq 7 \end{cases}$

Proof: Based on the vertices in cycle C_n, we have following cases.

Case (i) Let $n = 3$
Let C_3 be the cycle with 3 nodes. Let v_1, v_2, v_3be the vertices of cycle C_3. Let $E(C_3)$ be the edges of the cycle where $E(C_3) = \{v_1v_2, v_2v_3, v_3v_1\}$. Now, we switch any arbitrary vertex $v_i, 1 \leq i \leq 3$, without loss of generality, we consider vertex v_2 is switched, such that modified graph $C'_3$is obtained with vertices v_1, v_2, v_3 and edges$E(C'_n) = \{v_1v_3\}$. Degree of $v_1$and degree of $v_2$is 1and degree of $v_2 = 0$ (isolated vertex).

Assign the colorc_1 to the vertex v_1. Color c_2 be assigned to vertex v_2 and the vertices $v_3$will have color c_3. This coloring is proper. Vertices $\{v_1, v_3\}$ of graph C'_3 will power dominate either color class $c_1 = \{v_1\}$ or color class $c_3 = \{v_3\}$. The isolated vertex will power dominate itself by the definition. Therefore, every vertex in the graph C'_3 will power dominate atleast one color class. The power dominator chromatic number for the cycle C'_3 with 3 nodes obtained by a switching any arbitrary vertex in cycle $C_3$is 3 i.e., $\chi_{pd}(C'_3) = 3$.

Case (ii): Let $n = 4$
Let C_4 be the cycle with $n = 4$ nodes. Let the vertex v_1, v_2, v_3, v_4be the vertices of cycle C_4. Let $E\ (C_4) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.

Now, let us switch an arbitrary vertex, without loss of generality, we consider vertex $v_2$is switched such that modified graph $C'_4$is obtained with vertices$v_1, v_2, v_3, v_4$and $E(C'_4) = \{v_1v_4, v_2v_4, v_3v_4\}$. Degree of v_4 is 3 and degree of vertices v_1, v_2, v_3 is 1.

Assign the colorc_1 to the vertex v_1, v_2, v_3 . Color c_2 be assigned to vertex v_4. This coloring is proper. The vertices $\{v_1, v_2, v_3\}$ of graph C'_4 will power dominate color class $c_2 = \{v_4\}$. Vertex v_4 power dominate the vertices with color $C_1 = \{v_1, v_2, v_3\}$ and itself by definition. Therefore, every vertex in the graph C'_4 will power dominate atleast one color class. The power dominator chromatic number for the cycle C'_4 with 4 nodes obtained by a switching any arbitrary vertex with degree 2 in cycle $C_4$is 2 i.e., $\chi_{pd}(C'_4) = 2$.

Case (iii): Let $n = 5$
Let C_5 be the cycle with 5 nodes. Let v_1, v_2, v_3, v_4, v_5be the vertices of cycle C_5. Let $E(C_5)$ be the edges of the cycle where $E(C_5) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$. Now, let us switch any arbitrary vertex, without loss of generality, we consider vertex $v_2$with degree 2 was switched such that modified graph C'_5 was obtained with vertices v_1, v_2, v_3, v_4, v_5 and $E(C'_5) = \{v_1v_5, v_2v_5, v_2v_4, v_3v_4, v_4v_5\}$. Degree of v_2 is 2, degrees of vertices v_4, v_5 is 3, and vertices v_1 and v_3 has degree 1 (pendent vertices)

The following method of assigning colors will confirm the power dominator coloring is proper. Assign the colorc_1 to the pendent vertices v_1, v_2 and vertex v_3. Color c_2 be assigned to vertex v_4 and the vertices $v_5$will have colorc_3. Thiscoloring is proper. Thus, the vertices $\{v_1, v_2, v_3\}$ of graph C'_5 will power dominate either color class $c_2 = \{v_4\}$ or color class $c_3 = \{v_5\}$. Vertex v_4 and vertec $v_5$will power dominate the verices with color $C_1 = \{v_1, v_2, v_3\}$ and itself by definition. Therefore, every vertex in the graph C'_5 will power dominate atleast one color class. The power dominator chromatic number for the cycle C_5' with 5 nodes obtained by a switching any arbitrary vertex in cycle $C_5$is 3 i.e., $\chi_{pd}(C_5') = 3$.

Case (iv): Let $n = 6$
Let C_6 be the cycle with 6 nodes. Let $v_1, v_2, v_3, v_4, v_5, v_6$be the vertices of cycle C_6. Let $E(C_6) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$.

Now,let us switch any arbitrary vertex, without loss of generality, we consider vertex $v_2$with degree 2 is switched such that modified graph C_6' was obtained with vertices $v_1, v_2, v_3, v_4, v_5, v_6$ and E$(C_5') = \{v_1v_6, v_2v_4, v_2v_5, v_2v_6, v_3v_4, v_4v_5, v_5v_6\}$. Degree of v_2 is 3, degrees of vertices v_4, v_5, v_6 is 3, and vertices v_1 and v_3 has degree 1 (pendent vertices)

Assign the colorc_1 to the pendent vertices v_1, v_2 and vertex v_3. Color c_2 be assigned to vertex v_4, vertex $v_5$will have colorc_3, vertex $v_6$will have colorc_4.This coloring is proper. Thus, the vertices $\{v_1, v_2, v_3\}$ of graph C_5' will power dominate either color class $c_2 = \{v_4\}$ and color class $c_3 = \{v_5\}$ and color class $c_4 = \{v_6\}$. Vertex v_4 will power dominate the vertex v_5 with color $C_3 = \{v_5\}$ and itself.Vertex v_5 will power dominate the vertex v_4 with color $C_2 = \{v_3\}$, vertex v_6 with color $C_4 = \{v_6\}$ and itself. Vertex v_6 will power dominate the vertex v_5 with color $C_3 = \{v_5\}$ and itself. Therefore, every vertex in the graph C_6' will power dominate atleast one color class. The power dominator chromatic number for the cycle C_6' with 6 nodes obtained by a switching any arbitrary vertex with degree 2 in cycle C_6 is 4 i.e., $\chi_{pd}(C_6') = 4$

Case (v) Let $n \geq 7$
Let C_n be the cycle with $n \geq 7$ nodes. Let $v_1, v_2, v_3 \dots, v_n$be the vertices of cycle C_n. Let $E(C_n) = \{v_iv_{i+1} \,/\, 1 \leq i \leq n-1\} \cup \{v_nv_1\}$. Now, let us switch any arbitrary vertex with degree 2, without loss of generality, we consider vertex $v_2$with degree 2 was switched such that cycle $C_n'$was obtained with vertices $v_1, v_2, v_3, v_4, \dots, v_n$ and edges$E(C_n') = \{v_iv_{i+1} \,/\, 3 \leq i \leq n-1\} \cup \{v_nv_1\} \cup \{v_2v_j \,/\, 4 \leq j \leq n\}$.Degree of vertices $v_j, 4 \leq j \leq n$ and degree of $v_2$is $n-3$, vertices v_1 and v_3 are pendent vertices (degree 1).

Assign the colorc_1 to vertices v_2. Color c_3 be assigned to vertex v_4, and the vertex v_n havecolorc_4. Color C_2 be assigned to all odd indexed vertices $\left\{v_{2i-1}/1 \leq i \leq \left\lceil\frac{n-1}{2}\right\rceil\right\}$. Color c_5 be assigned to all the even indexed vertices $\left\{v_{2i} \,/\, 3 \leq i \leq \left\lfloor\frac{n-1}{2}\right\rfloor\right\}$.

This coloring is proper. Thus, all the vertices $\{v_4, \dots, v_n\}$ of graph C_n' will power dominate color class $c_1 = \{v_2\}$. Vertex v_1 will power dominate vertex v_n with color c_4 and color class c_1 and vertex v_3, will power dominate vertex v_4 with color c_3 and color class c_1. Vertex $v_2$will power dominate color class $C_5 = \left\{v_{2i} \,/\, 3 \leq i \leq \left\lfloor\frac{n-1}{2}\right\rfloor\right\}$ and itself by the definition. Therefore, every vertex in the graph C_n' will power dominate atleast one color class. The power dominator chromatic number for the graph C_n'obtained by switching any arbitrary vertex with degree 2 in cycle C_n is 5.i.e., $\chi_{pd}(C_n') = 5$, if $n \geq 7$. □

Example 1: In figure2, the power dominator coloring of modified graph C_4' obtained by a switching a vertex v_2 in cycle $C_4$is shown

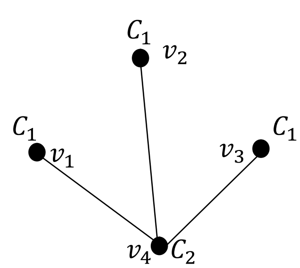


Fig.2 Modified graphC_4'on switching $v_2$of C_4

Theorem 8: For any $n \geq 3$,the power dominator chromatic number for modified graphDF_n', obtained by switching any apex vertex of double fan graph DF_n is 3.

Proof: Let DF_n be the double fan graph with vertex set $\{v_0, v_1, v_2, \dots v_n, v_0'\}$, where v_0 and v_0' being apex vertices and the vertices of path $P_n$being shared with apex vertices v_0 and v_0' are $\{v_1, v_2, \dots v_n\}$. Let$E(DF_n) = \{v_0 v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n\} \cup \{v_0' v_i / 1 \leq i \leq n\}$.

Without loss of generality, Let us switch an apex vertex v_0'in a double fan graph DF_n to u, such that new graph DF_n' was obtained with vertices $v_1, v_2, v_3, v_4, \dots, v_n, v_0'$ and edges$E(DF_n') = \{v_0 v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n\} \cup \{v_0' v_0\}$.

Assign the colorc_1 to the apex vertex v_0. The switched apex vertex v_0' is colored with c_2 and vertices $\{v_1, v_2, v_3, \dots, v_n\}$ of path P_n assigned color c_2 and c_3 alternatively.This coloring is proper. All the vertices $\{v_0', v_1, v_2, \dots, v_n\}$ of a path $P_n$will power dominate color class $c_1 = \{v_0\}$. And, the apex vertex $\{v_0\}$will power dominate either color class c_2 or color class c_3. Therefore, every vertex in the graph DF_n' will power dominate atleast one color class. The power dominator chromatic number for the graph DF_n'obtained by a switching any apex vertex in double fan graph DF_n is 3..i.e., $\chi_{pd}(DF_n') = 3$. □

Theorem 9:

For any $n \geq 3$,the power dominator chromatic number for modified graphDF_n', obtained by switching any arbitrary vertex in the path of double fan graph DF_n is 4.

Proof: Let DF_n be the double fan graph with vertex set $\{v_0, v_1, v_2, \dots v_n, v_0'\}$, where v_0 and v_0' being apex vertices and the vertices of path $P_n$being shared with apex vertices v_0 and v_0' are $\{v_1, v_2, \dots v_n\}$. Let $E(DF_n) = \{v_0 v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n\} \cup \{v_0' v_i / 1 \leq i \leq n\}$.

Without loss of generality, let us switch an arbitrary vertex $v_k (1 \leq k \leq n)$of path P_n in double fan graph DF_n, such that new graph DF_n' was obtained with vertices $v_1, v_2, v_3, v_4, \dots, v_n, v_0'$and $E(DF_n') = \{\{\{v_0 v_i / 1 \leq i \leq n, i \neq k\} \cup \{v_i v_{i+1} / 1 \leq i \leq n\} \cup \{v_0' v_i / 1 \leq i \leq n, i \neq k\}\} \cup \{v_k v_i / 1 \leq i \leq n, i \neq k-1 \,\&\, i \neq k+1\} - \{v_{k-1} v_k, v_k v_{k+1}\}\}$.

Assign the colorc_1 to the apex vertices v_0, v_0'. Color c_4 be given to vertex v_k. The vertices $\{v_1, v_2, v_3, \dots, v_{k-1}, v_{k+1}, \dots, v_n\}$ of path P_n assigned color c_2 and c_3 alternatively.This coloring is proper. All the vertices $\{v_1, v_2, \dots, v_{k-1}, v_{k+1}, \dots, v_n\}$ of a path $P_n$and will power dominate color class $c_1 = \{v_0', v_0\}$. And, all the apex vertices $\{v_0', v_0\}$will power dominate either color class c_2 or color class c_3. Vertex v_k ill power dominate itself. Therefore, every vertex in the graph DF_n' will power dominate atleast one color class. The power dominator chromatic number for the graph DF_n'obtained by a switching an arbitrary vertex in double fan graph DF_n is 3..i.e., $\chi_{pd}(DF_n') = 3$. □

CONCLUSION

On relating graph coloring problem with power dominating sets, power dominator coloring was introduced. The main objective of this paper is to find the power dominator chromatic number for the graphspath, cycle, complete graph, bipartite graph, double fan graph, with the context of vertex switching. There is scope for studying the properties of power dominator coloring for more graphs related to fan F_n.

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